

Math 1A Midterm 1 Review

- THERE WILL BE VERY FEW PROBLEMS THAT EXPLICITLY ASK YOU TO FIND THE LIMIT OF A FUNCTION. INSTEAD, YOU WILL BE ASKED TO SOLVE PROBLEMS WHERE YOU NEED TO WRITE YOUR OWN LIMITS, THEN FIND THEM.**
- IF YOU HAVE TAKEN DIFFERENTIAL CALCULUS BEFORE, DO NOT USE DIFFERENTIATION SHORTCUTS.**
- YOU SHOULD ONLY REQUIRE A CALCULATOR FOR QUESTIONS MARKED [C].**
- UNLESS A GRAPH IS GIVEN, YOU MUST BE ABLE TO SOLVE EACH PROBLEM WITHOUT A GRAPH.**

- [1][C] Estimate the slope of the tangent line to the curve $y = \sqrt{x + \sqrt{\cos x}}$ at the point $(0, 1)$ using the slopes of several secant lines.
- [2] The position of an object (in meters) at time t seconds, is given by the function $f(t) = t^2 \cos \pi t$. Find the average velocity of the object over the interval $[1, 5]$. Specify the units.
- [3] Sketch the graph of a function $f(x)$ which satisfies the following conditions:
 $\lim_{x \rightarrow -2^+} g(x) = -3$, $\lim_{x \rightarrow -2^-} g(x) = \infty$, $\lim_{x \rightarrow 1} g(x) = -\infty$, $\lim_{x \rightarrow -\infty} g(x) = 2$, and $\lim_{x \rightarrow \infty} g(x) = -2$
- [4] Prove that $\lim_{x \rightarrow 0} x^4 \cos \frac{1}{x^2} = 0$.
- [5] Let $f(x) = \begin{cases} 2x - 3 & \text{if } x < -1 \\ x^2 - 6 & \text{if } -1 < x < 2 \\ 4x - 6 & \text{if } x \geq 2 \end{cases}$.
- [a] Find $\lim_{x \rightarrow -2} f(x)$.
- [b] Find $\lim_{x \rightarrow -1} f(x)$.
- [c] Find $\lim_{x \rightarrow 2} f(x)$.
- [6] Find the value of a if $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + a} - 1}{x - 2} = 2$.
- [7] If $\lim_{x \rightarrow 2} f(x) = -3$ and $\lim_{x \rightarrow 2} g(x) = 4$, find $\lim_{x \rightarrow 2} \frac{x^2 g(x)}{1 + f(x)}$. Show clearly how the limit laws are used in your solution.
- [8] Find the discontinuities of $f(x) = \frac{x+2}{x^2-9}$, and find the one-sided limits at each discontinuity.
- [9] Let $f(x) = \begin{cases} 2x + a & \text{if } x < -1 \\ 3 - x & \text{if } -1 < x < 2 \\ bx - 1 & \text{if } x \geq 2 \end{cases}$.
- [a] Find the value of a so that $f(x)$ is continuous at $x = -1$.
- [b] Find the value of b so that $f(x)$ is continuous at $x = 2$.
- [c] If $a = 6$ and $b = 3$, find all discontinuities of $f(x)$ and find the type of each discontinuity (removable, jump or infinite).
- [10] Use the Intermediate Value Theorem to prove that the equation $\cos 2x = x^2$ has a solution in the interval $[0, \pi]$.
- [11] Find all horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{4+9x^2}}{2x-1}$.

[12] If $f(x) = x^3 - 3x + 2$, find $f'(-2)$ using both definitions of $f'(a)$.

[13] Find a function f and a number a such that the derivative of f at a is given by

[a] $\lim_{h \rightarrow 0} \frac{\cos(\pi(h-1)) + 1}{h}$

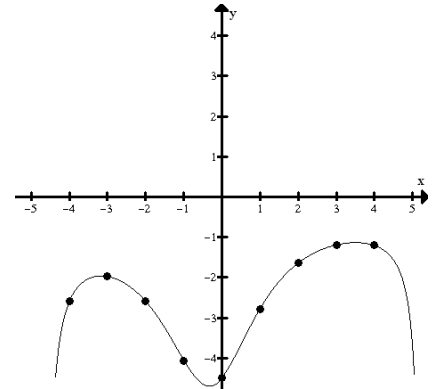
[b] $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x + 2}$

[14] The position of an object (in feet) at time t minutes, is given by the function $f(t) = \sqrt{t^2 - 5}$. Find the instantaneous velocity of the object at time $t = 3$. Specify the units.

[15] Find the equation of the tangent line to the curve of $f(x) = \frac{2x}{1-x}$ at $x = 2$.

[16] The graph of f is shown to the right. Arrange the following from least (most negative) to greatest (most positive).

$$\begin{array}{ccccccccc} 0 & f'(-4) & f'(-2) & f'(2) & f'(4) & & & & \\ \hline \text{LEAST} & & & & & & & & \text{GREATEST} \end{array}$$



[17] The time required to defrost a piece of frozen meat in the refrigerator depends on the temperature inside the refrigerator. Let $t = f(T)$, where t is the defrost time (in hours), and T is the refrigerator temperature (in $^{\circ}\text{C}$)

[a] Give the practical meaning (including units) of $f(4) = 6$.

[b] Give the practical meaning (including units) of $f'(4) = -1$.

[c] Is there a value of T_0 for which you would expect $f'(T_0) > 0$? Why or why not?

[18] Using the definition of the derivative, find the derivatives of the following functions.

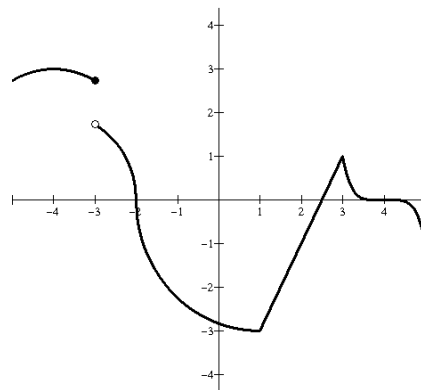
[a] $f(t) = \frac{1}{\sqrt{1-t}}$

[b] $g(x) = \frac{4x}{2-x}$

[19] The graph of $f(x)$ is shown on the right.

[a] Find all x -coordinates where $f'(x)$ is undefined, and explain briefly why.

[b] Sketch a graph of $f'(x)$.



[20] If the tangent line to the graph of $y = f(x)$ at $x = 4$ is $x - 2y = 6$, prove that $\lim_{x \rightarrow 4} f(x) = -1$.

YOU MUST ALSO KNOW THE FOLLOWING DEFINITIONS AND THEOREMS:

Definitions

- vertical/horizontal asymptote
- continuity at a point
- removable discontinuity (from lecture)
- jump discontinuity (from lecture)
- derivative at a point
- derivative function

Theorems

- Squeeze Theorem
- Intermediate Value Theorem
- Differentiability implies continuity