Math 1A (10:30am – 11:20am)
Midterm 2 Version B
Wed Feb 16, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER CALCULUS LEVEL ALGEBRAIC WORK AND USE PROPER NOTATION

Find $\frac{d^2}{dx^2}\frac{(3+x)^2}{\sqrt[4]{x}}$. **SIMPLIFY YOUR ANSWER, AND FACTOR.** **SCORE: _____ / 10 POINTS**

$$= \frac{d^2}{dx^2}\frac{9+6x+x^2}{x^{\frac{1}{4}}}$$

$$= \frac{d^2}{dx^2}\left(9x^{-\frac{1}{4}}+6x^{\frac{3}{4}}+x^{\frac{7}{4}}\right)$$

$$= \frac{d}{dx}\left(-\frac{9}{4}x^{-\frac{5}{4}}+\frac{9}{2}x^{-\frac{1}{4}}+\frac{7}{4}x^{\frac{3}{4}}\right)$$

$$= \frac{45}{16}x^{-\frac{9}{4}}-\frac{9}{8}x^{-\frac{5}{4}}+\frac{21}{16}x^{-\frac{1}{4}}$$

$$= \frac{3}{16}x^{-\frac{9}{4}}(15-6x+7x^2)$$

Find $\lim_{x\to\infty}\frac{4+5e^x}{3e^{2x}-1}$ using only the techniques discussed in chapter 2. **SCORE: _____ / 7 POINTS**

$$= \lim_{x\to\infty}\frac{4+5e^x}{3e^{2x}-1}\cdot\frac{e^{-2x}}{e^{-2x}}$$

$$= \lim_{x\to\infty}\frac{4e^{-2x}+5e^{-x}}{3-e^{-2x}} = \frac{4(0)+5(0)}{3-0} = 0$$

Find $\frac{d}{dx}5^{\sec^2 x}$. **SCORE: _____ / 7 POINTS**

$$= 5^{\sec^2 x}(\ln 5)(2\sec x)(\sec x\tan x)$$

$$= (2\ln 5)\,5^{\sec^2 x}\sec^2 x\tan x$$

Estimate the value of $\cot 0.8$ using a linear approximation chosen at an appropriate point. **SCORE: _____ / 15 POINTS**
Your final answer may involve e, π, **logarithms and/or radicals.**

$$\cot 0.8 \approx \cot\frac{\pi}{4}+\left(-\csc^2\frac{\pi}{4}\right)\left(0.8-\frac{\pi}{4}\right)$$

$$= 1-2\left(\frac{4}{5}-\frac{\pi}{4}\right)$$

$$= 1-\frac{8}{5}+\frac{\pi}{2}$$

$$= \frac{\pi}{2}-\frac{3}{5}$$

The position of an object at time t is given by $s(t) = \tan^{-1}(1+t^2)$. **SCORE: _____ / 22 POINTS**

Find the **acceleration** of the object at time $t = 1$.

$$s'(t) = \frac{1}{1+(1+t^2)^2} \cdot 2t = \frac{2t}{2+2t^2+t^4}$$

$$s''(t) = \frac{2(2+2t^2+t^4) - 2t(4t+4t^3)}{(2+2t^2+t^4)^2}$$

$$s''(1) = \frac{2(5)-2(8)}{5^2} = \frac{-6}{25}$$

Find the slope of the tangent line to $(1 + x^2 + xe^{2y})^4 = x^2 \cos y$ at $(-1, 0)$. **SCORE: _____ / 22 POINTS**

$$4(1+x^2+xe^{2y})^3\left(2x+e^{2y}+x(2e^{2y})\frac{dy}{dx}\right) = 2x\cos y - x^2 \sin y \frac{dy}{dx}$$

$$4(1)^3\left(-1-2\left.\frac{dy}{dx}\right|_{(-1,0)}\right) = -2$$

$$-1-2\left.\frac{dy}{dx}\right|_{(-1,0)} = -\frac{1}{2}$$

$$-2\left.\frac{dy}{dx}\right|_{(-1,0)} = \frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{(-1,0)} = -\frac{1}{4}$$

Show that $y = \ln(1+x^3)$ and $y = \dfrac{2-x^3}{6x}$ are orthogonal. **SCORE: _____ / 15 POINTS**

$$\frac{dy}{dx} = \frac{1}{1+x^3} \cdot 3x^2 = \frac{3x^2}{1+x^3}$$

$$\frac{dy}{dx} = \frac{-3x^2(6x)-(2-x^3)(6)}{(6x)^2} = \frac{-18x^3-12+6x^3}{36x^2} = \frac{-12x^3-12}{36x^2} = -\frac{x^3+1}{3x^2}$$

SLOPES ARE NEGATIVE RECIPROCALS

If the position of an object at time t is given by $s(t) = (\csc t)^{\arcsin t}$, find the velocity function. **SCORE: _____ / 15 POINTS**

$$\ln s(t) = \arcsin t \ln \csc t$$

$$\frac{1}{s(t)} s'(t) = \frac{1}{\sqrt{1-t^2}} \ln \csc t + \arcsin t \left(\frac{1}{\cancel{\csc t}}\right)(-\cancel{\csc t}\cot t)$$

$$s'(t) = s(t)\left(\frac{\ln \csc t}{\sqrt{1-t^2}} - \arcsin t \cot t\right)$$

$$= (\csc t)^{\arcsin t}\left(\frac{\ln \csc t}{\sqrt{1-t^2}} - \arcsin t \cot t\right)$$

Using the definition of the derivative, prove the derivative of $f(x) = \tan x$. **SCORE: _____ / 22 POINTS**

<u>You may use the two trigonometric limits proved in class, without reproving them. You MUST NOT use any differentiation shortcuts.</u>

$$f'(x) = \lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h)\cos x - \cos(x+h)\sin x}{h\cos(x+h)\cos x}$$

$$= \lim_{h \to 0} \frac{\sin(x+h-x)}{h\cos(x+h)\cos x}$$

$$= \lim_{h \to 0} \frac{\sin h}{h} \lim_{h \to 0} \frac{1}{\cos(x+h)\cos x} = 1 \cdot \sec^2 x = \sec^2 x$$

The line $y = 3x$ is tangent to a cubic function at the origin. The tangent lines at $x = \pm 1$ are both horizontal. **SCORE: _____ / 15 POINTS**

Find the equation of the tangent line to the cubic function at $x = 1$.

$f(x) = ax^3 + bx^2 + cx + d \qquad f'(x) = 3ax^2 + 2bx + c$

$f(0) = 0 \Rightarrow d = 0$

$f'(0) = 3 \Rightarrow c = 3$

$f'(1) = 0 \Rightarrow 3a + 2b + 3 = 0$

$f'(-1) = 0 \Rightarrow 3a - 2b + 3 = 0$

$6a + 6 = 0$

$a = -1$

$b = 0$

$f(x) = -x^3 + 3x$

$f(1) = 2$

$y - 2 = 0(x-1)$

$y = 2$

BONUS POINTS ON OTHER SIDE ➡