Math 1A (7:30am – 8:20am)
Midterm 2 Version A
Wed Feb 16, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER CALCULUS LEVEL ALGEBRAIC WORK AND USE PROPER NOTATION

Find $\dfrac{d^2}{dx^2}\dfrac{(4+x)^2}{\sqrt[3]{x}}$. **SIMPLIFY YOUR ANSWER, AND FACTOR.** **SCORE: _____ / 10 POINTS**

$$= \frac{d^2}{dx^2}\frac{16+8x+x^2}{x^{\frac{1}{3}}}$$

$$= \frac{d^2}{dx^2}\left(16x^{-\frac{1}{3}}+8x^{\frac{2}{3}}+x^{\frac{5}{3}}\right) = \frac{d}{dx}\left(-\frac{16}{3}x^{-\frac{4}{3}}+\frac{16}{3}x^{-\frac{1}{3}}+\frac{5}{3}x^{\frac{2}{3}}\right)$$

$$= \frac{64}{9}x^{-\frac{7}{3}}-\frac{16}{9}x^{-\frac{4}{3}}+\frac{10}{9}x^{-\frac{1}{3}}$$

$$= \frac{2}{9}x^{-\frac{7}{3}}\left(32-8x+5x^2\right)$$

Show that $3x^2+y^2=a$ and $x=by^3$ are orthogonal trajectories (where a and b are constants). **SCORE: _____ / 15 POINTS**
NOTE: DO NOT SOLVE EXPLICITLY FOR y.

$$6x+2y\frac{dy}{dx}=0 \qquad 1=3by^2\frac{dy}{dx}$$

$$\frac{dy}{dx}=-\frac{6x}{2y}=-\frac{3x}{y} \qquad \frac{dy}{dx}=\frac{1}{3by^2}$$

$$-\frac{\cancel{3}x}{y}\cdot\frac{1}{\cancel{3}by^2}=-\frac{x}{by^3}=-\frac{x}{x}=-1$$

The total amount you spend on large repairs to your house depends on how much you spend on regular maintenance each year. If $A=R(m)$, where m is the amount you spend on maintenance each year (in hundreds of dollars), and A is the total amount you spend on large repairs (in thousands of dollars), what does the statement $f'(5)=-15$ mean ? Give the units of measurement for each number in your answer. **SCORE: ___ / 7 POINTS**
NOTE: Your answer should NOT include "derivative", "instantaneous", "rate of change", "with respect to", "slope" or "tangent line".

IF YOU SPEND $500 ON MAINTENANCE EACH YEAR,
YOU WILL SPEND $15,000 LESS ON LARGE REPAIRS
FOR EACH ADDITIONAL $100 YOU SPEND ON MAINTENANCE
EACH YEAR.

If $q(t) = (\arctan t)^{\csc t}$, find $q'(t)$. **SCORE: _____ / 15 POINTS**

$y = (\arctan t)^{\csc t}$

$\ln y = \csc t \ln \arctan t$

$\frac{1}{y}\frac{dy}{dt} = -\csc t \cot t \ln \arctan t + \csc t \frac{1}{\arctan t}\frac{1}{1+t^2}$

$\frac{dy}{dt} = y\left(-\csc t \cot t \ln \arctan t + \frac{\csc t}{(1+t^2)\arctan t}\right)$

$q'(t) = (\arctan t)^{\csc t} \csc t \left(\frac{1}{(1+t^2)\arctan t} - \cot t \ln \arctan t\right)$

Estimate the value of $\sec 0.6$ using a linear approximation chosen at an appropriate point. **SCORE: _____ / 22 POINTS**

<u>Your final answer may involve</u> e, π, **<u>logarithms and/or radicals.</u>**

$\sec 0.6 \approx \sec\frac{\pi}{6} + \left(\sec\frac{\pi}{6}\tan\frac{\pi}{6}\right)\left(0.6 - \frac{\pi}{6}\right)$

$= \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{3}\left(\frac{3}{5} - \frac{\pi}{6}\right)$

$= \frac{2}{\sqrt{3}} + \frac{2}{3}\left(\frac{3}{5} - \frac{\pi}{6}\right)$

$= \frac{2}{\sqrt{3}} + \frac{2}{5} - \frac{\pi}{9}$

The position of an object at time t is given by $s(t) = \ln(1+3t^4)$. **SCORE: _____ / 22 POINTS**

Find the **<u>acceleration</u>** of the object at time $t = 1$.

$s'(t) = \frac{1}{1+3t^4} \cdot 12t^3 = \frac{12t^3}{1+3t^4}$

$s''(t) = \frac{36t^2(1+3t^4) - 12t^3(12t^3)}{(1+3t^4)^2}$

$s''(1) = \frac{36(4) - 12(12)}{4^2} = 0$

The limit $\lim_{h\to 0}\frac{(h-2)e^{4-(h-2)^2}+2}{h}$ is the derivative of some function $f(x)$ at some point $x=a$. **SCORE: _____ / 22 POINTS**

Find the function, the point, and the value of the limit.

$f(a+h) = (h-2)e^{4-(h-2)^2}$ LOOKS LIKE $a=-2$

$f(-2+h) = f(h-2) = (h-2)e^{4-(h-2)^2}$ LOOKS LIKE $f(x) = xe^{4-x^2}$

CHECK: $f'(-2) = \lim_{h\to 0}\frac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0}\frac{(-2+h)e^{4-(-2+h)^2} - -2}{h}$

$= \lim_{h\to 0}\frac{(h-2)e^{4-(h-2)^2}+2}{h}$

$f'(x) = e^{4-x^2} + xe^{4-x^2}(-2x)$

$f'(-2) = 1 + (-2)(4) = -7$

Find the equation of the normal line to $y = 2^{\tan x} + \arccos x$ at $x=0$. **SCORE: _____ / 15 POINTS**

<u>Your final answer may involve</u> e, π, **<u>logarithms and/or radicals.</u>**

$\frac{dy}{dx} = 2^{\tan x}(\ln 2)\sec^2 x - \frac{1}{\sqrt{1-x^2}}$

$\left.\frac{dy}{dx}\right|_{x=0} = \ln 2 - 1$

$y(0) = 1 + \frac{\pi}{2}$

$y - (1+\frac{\pi}{2}) = \frac{1}{1-\ln 2}(x-0)$

$y = 1 + \frac{\pi}{2} + \frac{1}{1-\ln 2}x$

Using the definition of the derivative, prove the derivative of $f(x) = \cot x$. **SCORE: _____ / 22 POINTS**

<u>You may use the two trigonometric limits proved in class, without reproving them. You MUST NOT use any differentiation shortcuts.</u>

$f'(x) = \lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h}$

$= \lim_{h\to 0}\frac{\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x}}{h}$

$= \lim_{h\to 0}\frac{\cos(x+h)\sin x - \sin(x+h)\cos x}{h\sin(x+h)\sin x}$

$= \lim_{h\to 0}\frac{\sin(x-(x+h))}{h\sin(x+h)\sin x}$

$= \lim_{h\to 0}\frac{\sin(-h)}{h\sin(x+h)\sin x}$

$= \lim_{h\to 0}\frac{-\sin h}{h\sin(x+h)\sin x}$

$= \lim_{h\to 0}\frac{\sin h}{h}\cdot\lim_{h\to 0}\frac{-1}{\sin(x+h)\sin x}$

$= 1\cdot -\csc^2 x$

$= -\csc^2 x$

BONUS POINTS ON OTHER SIDE ➡