Math 1A
Midterm 2 Review

You should be able to find any derivative from this chapter.

3.1 3-32
3.2 3-30
3.3 1-16
3.4 7-50
3.5 5-20, 25-30, 45-54
3.6 2-30, 37-50
3.REV 1-42, 44, 46, 49-50

Knowing how to find derivatives is not enough, because once again, there will be very few questions which simply ask you to find a derivative. You should also be able to solve all the following types of problems.

[1] Estimate $\csc 0.5$ using a linear approximation chosen at an appropriate point.

[2] If $y = \frac{1}{x^2}$, find dx, Δy and dy if $x = 2$ and $\Delta x = 0.5$.

[3] Find $\frac{d^3}{dx^3} \sec x$. **Simplify your answer.**

[4] The position of an object at time t is given by the function $s(t) = \frac{2t^3 + 4t^2 - 3}{\sqrt{t}}$ for $t > 0$.

[a] Find the velocity of the object at time $t = 1$.

[b] Find the acceleration function. **Simplify and factor your answer.**

[5] Find the equations of the tangent lines to the curve $y = 1 + x^3$ that are perpendicular to $x + 12y = 1$.

[6] The line $y = 3x - 4$ is tangent to a quadratic function at the point $(1, -1)$. Find the equation of the tangent line to the quadratic function at $(2, 4)$.

[7] If $f(x) = \frac{x^3}{1 + x^2}$, find $f''(1)$.

[8] The following table gives values and derivatives of two functions at various inputs.

x	–3	–2	–1	0	1	2	3	4
$f(x)$	–2	0	2	4	–3	–1	1	3
$f'(x)$	4	–1	–3	2	–4	3	–2	1
$g(x)$	–1	1	3	–3	4	–2	0	2
$g'(x)$	2	4	–4	–1	3	1	–3	–2

[a] If $k(x) = x^3 f(x)$, find the equation of the tangent line to $y = k(x)$ at $x = 2$.

[b] If $j(x) = \frac{x^2}{f(x)}$, find the equation of the tangent line to $y = j(x)$ at $x = -1$.

[c] If $m(x) = \tan^{-1}(g(x))$, find the equation of the tangent line to $y = m(x)$ at $x = -3$.

[d] If $n(x) = g(f(x))$, find the equation of the tangent line to $y = n(x)$ at $x = 4$.

[9] If $h(x) = f(x)g(x)$, find formulae for $h''(x)$ and $h'''(x)$. Based on your answers, guess a formula for $h^{(4)}(x)$ (the fourth derivative of $h(x)$.

[10] Find all x-coordinates in the interval $[0, 2\pi]$ where the tangent line to $f(x) = 4x - 3\tan x$ is horizontal.

[11] If $f(x) = xg(x^2)$, find a formula for $f''(x)$. Your answer may involve g, g' and/or g''.

[12] Find the equation of the tangent line to $(1 + x^2 y^3)^5 = x^4 e^y$ at $(-1, 0)$.

[13] Show that $y = ax^4$ and $x^2 + 4y^2 = b$ are orthogonal trajectories. **See section 3.5, questions 59-62.**

[14] If $y = (\sin x)^{\frac{1}{x}}$, find $\frac{dy}{dx}$.

[15] The limit $\lim_{h \to 0} \frac{(h-1)e^{1-h} + e}{h}$ is the derivative of some function $f(x)$ at some point $x = a$. Find the function, the point, and the value of the limit.

[16] Prove that $(\csc x)' = -\csc x \cot x$ using the definition of the derivative.
Do not use the product, quotient or chain rules, nor the derivative of $\sin x$ **.**

You must also know the following definitions, theorems and proofs.

Definition e

Proofs derivatives of $\sin x$, $\cos x$, $\tan x$, $\csc x$, $\sec x$ and $\cot x$ (see [16] above)
using the definition of the derivative, without using the derivatives of any other trigonometric function
you may use the limits $\lim_{h \to 0} \frac{\sin h}{h} = 1$ and $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$ without proving them

derivatives of $\tan x$, $\csc x$, $\sec x$ and $\cot x$
using the quotient rule with the derivatives of $\sin x$ and $\cos x$

derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, and $\ln x$
using implicit differentiation with the derivatives of $\sin x$, $\cos x$, $\tan x$ and e^x