Math 1A (10:30am – 11:20am)
Midterm 3 Version A
Tue Mar 15, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER CALCULUS LEVEL ALGEBRAIC WORK AND USE PROPER NOTATION

Let f be a polynomial function such that $f'(x) = (3+x)^7(4-x)^8$. **SCORE: _____ / 10 POINTS**

[a] Find the critical numbers of f.

f' EXISTS FOR ALL X

$f' = 0$ IF $X = -3, 4$

[b] Classify each critical number as a local maximum, a local minimum or neither.

		-3		4	
f'	$-$		$+$		$+$
$(3+x)^7$	$-$		$+$		$+$
$(4-x)^8$	$+$		$+$		$+$

$X = -3$ IS A LOCAL MIN

4 NEITHER

Consider the function $f(x) = x^{-2}$ on the interval $[-2, 1]$. **SCORE: _____ / 15 POINTS**

[a] Does this situation satisfy the **conclusion** of the Mean Value Theorem ? Why or why not ?

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$-2c^{-3} = \frac{1-\frac{1}{4}}{1--2} = \frac{1}{4}$$

$$-\frac{2}{c^3} = \frac{1}{4} \longrightarrow c^3 = -8 \longrightarrow c = -2 \notin (-2, 1)$$

DOES NOT SATISFY CONCLUSION

[b] Does the Mean Value Theorem apply to this situation ? Why or why not ?

NO. f IS NOT CONT. @ $X = 0 \in [-2, 1]$

Let f be a continuous function with critical numbers 2 and 4 such that $f''(x) = -3x^2 + 20x - 32$. **SCORE: _____ / 10 POINTS**
Determine what the Second Derivative Test tells you about each critical number.

$f''(2) < 0$ LOCAL MAX

$f''(4) = 0$ NO CONCLUSION

Find $\lim_{x\to 0}(\cos x)^{\frac{1}{x^3}}$. 1^{∞} **SCORE: _____ / 15 POINTS**

Your final answer should be a number, ∞ or $-\infty$. **Write DNE only if none of the other answers apply.**

$\lim_{x\to 0} \ln(\cos x)^{\frac{1}{x^3}}$

$= \lim_{x\to 0} \frac{1}{x^3}\ln\cos x$

$= \lim_{x\to 0} \frac{\ln\cos x}{x^3}$ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{\frac{1}{\cos x}\cdot -\sin x}{3x^2}$

$= \lim_{x\to 0} \frac{-\tan x}{3x^2}$ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{-\sec^2 x}{6x}$ $\frac{-1}{0^{\pm}}$

$= \pm\infty$

IF $\ln(\cos x)^{\frac{1}{x^3}} \to \infty$

$(\cos x)^{\frac{1}{x^3}} \to \infty$

IF $\ln(\cos x)^{\frac{1}{x^3}} \to -\infty$

$(\cos x)^{\frac{1}{x^3}} \to 0$

SO, $\lim_{x\to 0} \ln(\cos x)^{\frac{1}{x^3}}$ DNE

Find the global extrema of $f(x) = x(x-10)^{\frac{2}{3}}$ on $[9, 18]$. **SCORE: _____ / 25 POINTS**

$f'(x) = (x-10)^{\frac{2}{3}} + \frac{2}{3}x(x-10)^{-\frac{1}{3}}$ DNE @ $x=10$

$= \frac{1}{3}(x-10)^{-\frac{1}{3}}(3(x-10)+2x)$

$= \frac{1}{3}(x-10)^{-\frac{1}{3}}(5x-30) = 0$ @ $x = 6 \notin [9,18]$

$f(9) = 9(-1)^{\frac{2}{3}} = 9$

$f(10) = 10(0)^{\frac{2}{3}} = 0$ ⟵ MIN

$f(18) = 18(8)^{\frac{2}{3}} = 18(4) = 72$ ⟵ MAX

Graph $f(x) = \dfrac{-3+\sqrt{x}}{x}$ using the procedure discussed in class. **SCORE: _____ / 25 POINTS**

CHECKLIST: (Check off as you finish finding these)

[__]	**Domain**	[__]	**Intercepts**	[__]	**Discontinuities (& behavior at)**
[__]	**Asymptotes**	[__]	**Intervals of increase/decrease**	[__]	**Intervals of upward/downward concavity**
[__]	**Horizontal/vertical tangent lines**	[__]	**Local extrema**	[__]	**Inflection points**

DOMAIN: $x > 0$

DISCONTINUITIES: $x \le 0$

X-INT: $-3+\sqrt{x} = 0 \rightarrow x = 9$

y-INT: NONE

$\lim_{x \to 0^+} \frac{-3+\sqrt{x}}{x} = -\infty \left(\frac{-3}{0^+}\right)$ V.A. @ $x = 0$

$\lim_{x \to \infty} \frac{-3+\sqrt{x}}{x} = \lim_{x \to \infty} (-3x^{-1} + x^{-\frac{1}{2}}) = 0 - 0 = 0$

OR $\left(\frac{\infty}{\infty}\right)$ $\lim_{x \to \infty} \frac{\frac{1}{2\sqrt{x}}}{1} = 0$

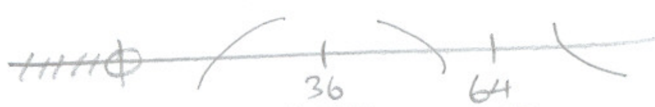
$f'(x) = 3x^{-2} - \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2}x^{-2}(6 - x^{\frac{1}{2}})$

$f''(x) = -6x^{-3} + \frac{3}{4}x^{-\frac{5}{2}} = \frac{3}{4}x^{-3}(-8 + x^{\frac{1}{2}})$

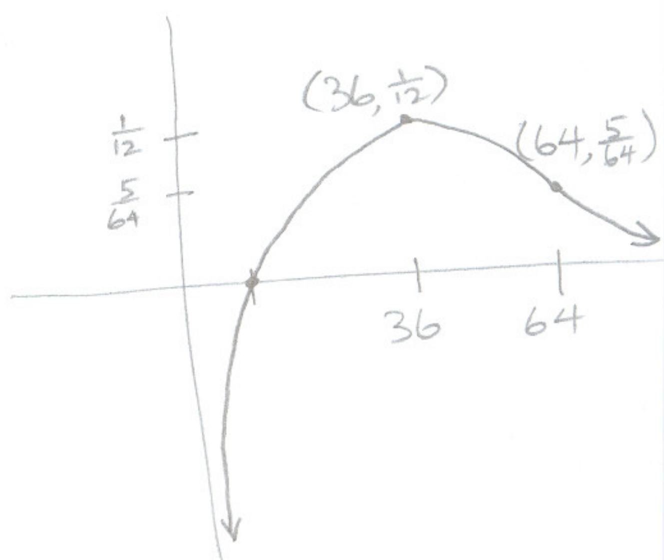
$f' = 0$ @ $x = 36$, DNE @ $x \le 0 \notin$ DOMAIN

$f'' = 0$ @ $x = 64$, DNE @ $x \le 0 \notin$ DOMAIN

f'	+	−	−
f''	−	−	+

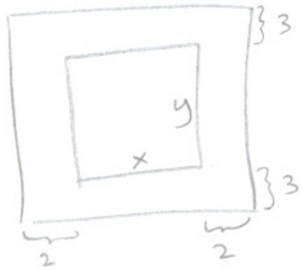


$\frac{1}{2}x^{-2}$	+	+	+
$6 - x^{\frac{1}{2}}$	+	−	−
$\frac{3}{4}x^{-3}$	+	+	+
$-8 + x^{\frac{1}{2}}$	−	−	+



The top and bottom margins of a poster are each 3 ft and the side margins are each 2 ft. **SCORE: _____ / 25 POINTS**

If the area of printed material on the poster is fixed at 24 ft^2, find the dimensions of the poster with the smallest total area.



$xy = 24$

$y = \frac{24}{x}$

MINIMIZE A = TOTAL AREA

$A = (x+4)(y+6)$

$= (x+4)(\frac{24}{x}+6)$

$= 24 + 6x + \frac{96}{x} + 24$

$= 48 + 6x + \frac{96}{x}$ $\quad x \in (0, \infty)$

$A' = 6 - \frac{96}{x^2}$ DNE @ $x=0 \notin$ DOMAIN

$= \frac{6x^2-96}{x^2} = 0$ @ $x = 4$

$A(4) = 8 \cdot 12 = 96$

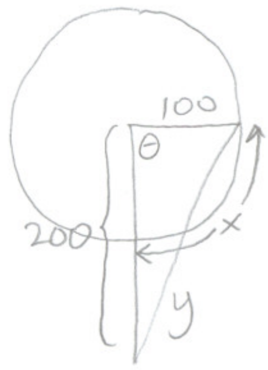
$\lim_{x \to 0^+} (x+4)(\frac{24}{x}+6) = \infty$ $\quad (4 \cdot \infty)$

$\lim_{x \to \infty} (x+4)(\frac{24}{x}+6) = \infty$ $\quad (\infty \cdot 6)$

THE SMALLEST POSTER IS 8 FT BY 12 FT

A runner sprints around a circular track of radius 100 m at a constant speed of 10 m/s. **SCORE: _____ / 25 POINTS**

The runner's friend is standing at a distance of 200 m from the center of the track. How fast is the distance between the friends changing when the distance between them is $100\sqrt{5}$ m ?



$\frac{d\theta}{dt} = \frac{1}{10}$ r/s

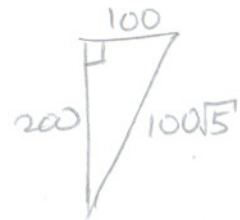
WANT $\frac{dy}{dt}$ WHEN $y = 100\sqrt{5}$ m

$y^2 = 100^2 + 200^2 - 2(100)(200)\cos\theta$

$2y\frac{dy}{dt} = 40000\sin\theta\frac{d\theta}{dt}$

$100\sqrt{5}\frac{dy}{dt} = 20000(1)\left(\frac{1}{10}\right)$

$\frac{dy}{dt} = \frac{20}{\sqrt{5}} = 4\sqrt{5}$ m/s



$x = r\theta$

$10\text{m} = 100\text{m}\cdot\theta$

$\theta = \frac{1}{10}$ r