Math 1A (7:30am – 8:20am)
Midterm 3 Version A
Tue Mar 15, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER CALCULUS LEVEL ALGEBRAIC WORK AND USE PROPER NOTATION

Let f be a polynomial function such that $f'(x) = (1+x)^8(2-x)^7$. **SCORE: _____ / 10 POINTS**

[a] Find the critical numbers of f.

f' EXISTS FOR ALL x

$f' = 0$ IF $x = -1, 2$

[b] Classify each critical number as a local maximum, a local minimum or neither.

	$x<-1$	$-1<x<2$	$x>2$
f'	+	+	−
$(1+x)^8$	+	+	+
$(2-x)^7$	+	+	−

$x = 2$ IS A LOCAL MAX

-1 NEITHER

Consider the function $f(x) = x^{-2}$ on the interval $[-1, 2]$. **SCORE: _____ / 15 POINTS**

[a] Does this situation satisfy the **conclusion** of the Mean Value Theorem ? Why or why not ?

$f'(c) = \frac{f(b)-f(a)}{b-a}$

$-2c^{-3} = \frac{\frac{1}{4}-1}{2-{-1}} = -\frac{1}{4}$

$-\frac{2}{c^3} = -\frac{1}{4} \longrightarrow c^3 = 8 \longrightarrow c = 2 \notin (-1, 2)$

DOES NOT SATISFY CONCLUSION

[b] Does the Mean Value Theorem apply to this situation ? Why or why not ?

NO. f IS NOT CONT. @ $x = 0 \in [-1, 2]$

Let f be a continuous function with critical numbers 2 and 4 such that $f''(x) = 3x^2 - 16x + 20$. **SCORE: _____ / 10 POINTS**
Determine what the Second Derivative Test tells you about each critical number.

$f''(2) = 0$ NO CONCLUSION

$f''(4) > 0$ LOCAL MIN

$\cup$

Find $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$. 1^{∞}

SCORE: _____ / 15 POINTS

Your final answer should be a number, ∞ or $-\infty$. **<u>Write DNE only if none of the other answers apply.</u>**

$$\lim_{x\to 0} \ln(\cos x)^{\frac{1}{x^2}}$$

$$= \lim_{x\to 0} \frac{1}{x^2} \ln \cos x$$

$$= \lim_{x\to 0} \frac{\ln \cos x}{x^2} \quad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\cos x} \cdot -\sin x}{2x}$$

$$= \lim_{x\to 0} \frac{-\tan x}{2x} \quad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{-\sec^2 x}{2} = -\frac{1}{2}$$

$$\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} = e^{-\frac{1}{2}}$$

Find the global extrema of $f(x) = x(x-10)^{\frac{2}{3}}$ on $[2, 11]$.

SCORE: _____ / 25 POINTS

$$f'(x) = (x-10)^{\frac{2}{3}} + \frac{2}{3}x(x-10)^{-\frac{1}{3}} \quad \text{DNE @ } x=10$$

$$= \frac{1}{3}(x-10)^{-\frac{1}{3}}(3(x-10)+2x)$$

$$= \frac{1}{3}(x-10)^{-\frac{1}{3}}(5x-30) = 0 \quad @\ x=6$$

$$f(2) = 2(-8)^{\frac{2}{3}} = 2(4) = 8$$

$$f(6) = 6(-4)^{\frac{2}{3}} = 6\sqrt[3]{16} > 6(2) = 12 \longleftarrow \text{MAX}$$

$$f(10) = 10(0)^{\frac{2}{3}} = 0 \longleftarrow \text{MIN}$$

$$f(11) = 11(1)^{\frac{2}{3}} = 11$$

Graph $f(x)=\dfrac{1-\sqrt{x}}{x}$ using the procedure discussed in class. **SCORE: _____ / 25 POINTS**

CHECKLIST: (Check off as you finish finding these)

[__]	**Domain**	[__]	**Intercepts**	[__]	**Discontinuities (& behavior at)**
[__]	**Asymptotes**	[__]	**Intervals of increase/decrease**	[__]	**Intervals of upward/downward concavity**
[__]	**Horizontal/vertical tangent lines**	[__]	**Local extrema**	[__]	**Inflection points**

DOMAIN : $x > 0$

DISCONTINUITIES : $x \leq 0$

X-INT : $1-\sqrt{x}=0 \Rightarrow x=1$

Y-INT : NONE

$\lim\limits_{x\to 0^+} \dfrac{1-\sqrt{x}}{x} = \infty \quad \left(\dfrac{1}{0^+}\right)$ V.A. @ $x=0$

$\lim\limits_{x\to\infty} \dfrac{1-\sqrt{x}}{x} = \lim\limits_{x\to\infty}(x^{-1}-x^{-\frac{1}{2}}) = 0-0=0$ OR $\lim\limits_{x\to\infty}\dfrac{1-\sqrt{x}}{x} = \lim\limits_{x\to\infty}\dfrac{-\frac{1}{2\sqrt{x}}}{1}=0$ $\left(\frac{-\infty}{\infty}\right)$

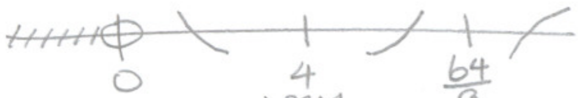
$f'(x) = -x^{-2}+\frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2}x^{-2}(-2+x^{\frac{1}{2}})$

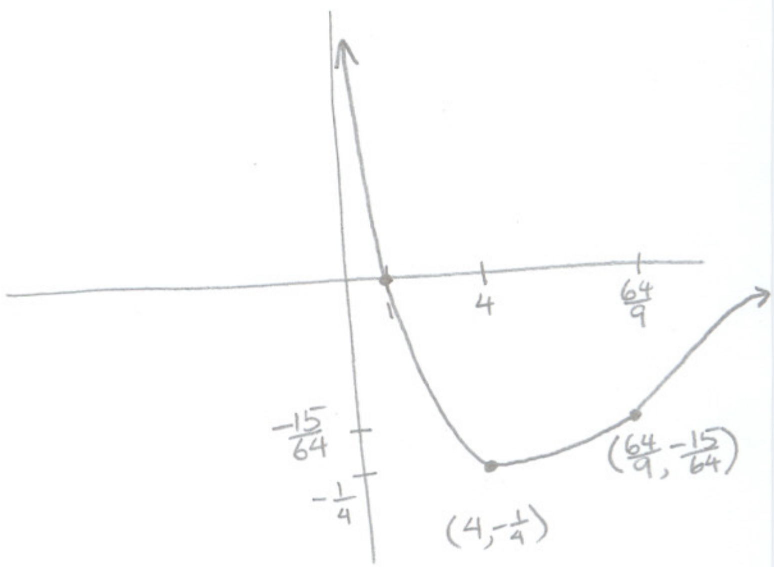
$f''(x) = 2x^{-3}-\frac{3}{4}x^{-\frac{5}{2}} = \frac{1}{4}x^{-3}(8-3x^{\frac{1}{2}})$

$f'=0$ @ $x=4$, DNE @ $x\leq 0 \notin$ DOMAIN

$f''=0$ @ $x=\frac{64}{9}$, DNE @ $x \leq 0 \notin$ DOMAIN

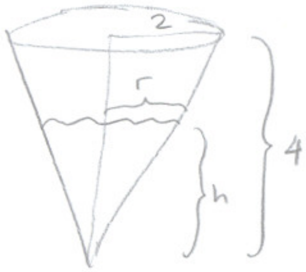
	$(0,4)$	4	$(4,\frac{64}{9})$	$\frac{64}{9}$	$(\frac{64}{9},\infty)$
f'	−		+		+
f''	+		+		−
		LOCAL MIN		I.P.	
$\frac{1}{2}x^{-2}$	+		+		+
$-2+x^{\frac{1}{2}}$	−		+		+
$\frac{1}{4}x^{-3}$	+		+		+
$8-3x^{\frac{1}{2}}$	+		+		−





A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. **SCORE: _____ / 25 POINTS**

If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.



$$\frac{r}{h} = \frac{2}{4}$$

$$r = \frac{1}{2}h$$

$$\frac{dV}{dt} = 2 m^3/min$$

WANT $\frac{dh}{dt}$ WHEN $h = 3m$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

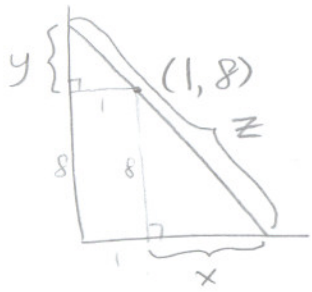
$$= \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{\cancel{12}_{4}} h^2 \frac{dh}{dt}$$

$$2 m^3/min = \frac{\pi}{4}(3m)^2 \frac{dh}{dt}$$

$$\frac{8}{9\pi} m/min = \frac{dh}{dt}$$

Find the length of the shortest line segment that is cut off by the first quadrant and passes through $(1, 8)$. **SCORE: _____ / 25 POINTS**



$$\frac{y}{1} = \frac{8}{x}$$

$$y = \frac{8}{x}$$

MINIMIZE $z^2 = (x+1)^2 + (y+8)^2$

$$Z = z^2 = (x+1)^2 + \left(\frac{8}{x}+8\right)^2 \qquad x \in (0, \infty)$$

$$Z' = 2(x+1) + 2\left(\frac{8}{x}+8\right)\left(-\frac{8}{x^2}\right)$$

DNE @ $x=0$ $\notin$ DOMAIN

$$= 2\left[x+1 - \frac{64}{x^3} - \frac{64}{x^2}\right]$$

$$= \frac{2}{x^3}(x^4 + x^3 - 64x - 64)$$

$$= \frac{2}{x^3}(x^3(x+1) - 64(x+1))$$

$$= \frac{2}{x^3}(x+1)(x^3-64)$$

$= 0$ @ $x = -1$ $\notin$ DOMAIN

AND $x = 4$

$$Z(4) = 5^2 + 10^2 = 125$$

$$\lim_{x\to 0^+}\left[(x+1)^2 + \left(\frac{8}{x}+8\right)^2\right] = \infty \qquad (1+\infty)$$

$$\lim_{x\to \infty}\left[(x+1)^2 + \left(\frac{8}{x}+8\right)^2\right] = \infty \qquad (\infty+64)$$

THE SHORTEST LINE SEGMENT IS $\sqrt{125} = 5\sqrt{5}$ UNITS LONG