

SCORE: \_\_\_\_ / 30 POINTS

**NO CALCULATORS ALLOWED**

**SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION**

**YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS  
 UNLESS SPECIFICALLY ASKED FOR**

Let  $f(x) = \begin{cases} 2x+15, & \text{if } x < -2 \\ 3-4x, & \text{if } -2 \leq x < 1. \\ x-2, & \text{if } x > 1 \end{cases}$

SCORE: \_\_\_\_ / 7 POINTS

[a] Is  $f(x)$  continuous at  $x = 1$  ?

If yes, show that all three conditions of continuity are satisfied. If no, show that at least one condition is not satisfied.

NO.  $f(1)$  DNE.

[b] Is  $f(x)$  continuous at  $x = -2$  ?

If yes, show that all three conditions of continuity are satisfied. If no, show that at least one condition is not satisfied.

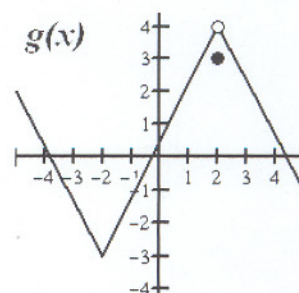
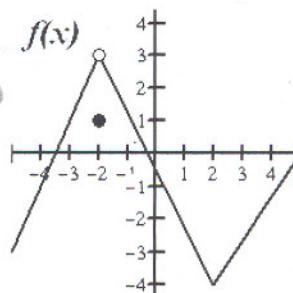
YES.  
 $\frac{1}{2}$   $f(-2) = 2(-2) + 15 = 11$   
 $\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} (2x + 15) = 11$   
 $\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^+} (3 - 4x) = 11$   
 $\frac{1}{2}$   $\lim_{x \rightarrow -2} f(x) = 11 = f(-2)$

The graphs of  $f$  and  $g$  are shown on the right.

SCORE: \_\_\_\_ / 5 POINTS

Find  $\lim_{x \rightarrow -2} [xf(x) - 5g(x)]$ , showing the proper use of the limit laws to justify your answer.

$= \lim_{x \rightarrow -2} [xf(x)] - \lim_{x \rightarrow -2} [5g(x)]$   
 $= \lim_{x \rightarrow -2} x \lim_{x \rightarrow -2} f(x) - \lim_{x \rightarrow -2} 5 \lim_{x \rightarrow -2} g(x)$   
 $= (-2)(3) - (5)(-3)$   
 $= \underline{9}$



State the complete definition of "vertical asymptote".

SCORE: \_\_\_ / 2 POINTS

$f$  HAS A VERTICAL ASYMPTOTE AT  $a$

IF  $\lim_{x \rightarrow a^+} f(x) = \infty$  OR  $\lim_{x \rightarrow a^-} f(x) = \infty$  OR  $\lim_{x \rightarrow a^+} f(x) = -\infty$   
OR  $\lim_{x \rightarrow a^-} f(x) = -\infty$

State the Squeeze Theorem.

SCORE: \_\_\_ / 2 POINTS

IF  $f(x) \leq g(x) \leq h(x)$  IN AN OPEN INTERVAL AROUND  $a$   
(EXCEPT POSSIBLY AT  $a$ )

AND  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$  THEN  $\lim_{x \rightarrow a} g(x) = L$

Evaluate the following limits.

SCORE: \_\_\_ / 14 POINTS

The answer should be a number,  $\infty$  or  $-\infty$ . Write DNE only if the other possibilities do not apply.

[a]  $\lim_{t \rightarrow 3} \frac{t-3}{2-\sqrt{t^2-5}}$   $\frac{0}{0}$

$= \lim_{t \rightarrow 3} \frac{(t-3)(2+\sqrt{t^2-5})}{4-(t^2-5)}$

$= \lim_{t \rightarrow 3} \frac{(t-3)(2+\sqrt{t^2-5})}{9-t^2}$

$= \lim_{t \rightarrow 3} \frac{(t-3)(2+\sqrt{t^2-5})}{(3-t)(3+t)}$

1 FOR FACTORING DENOMINATOR

1 FOR CANCELLING

$= -\frac{4}{6}$

$= -\frac{2}{3}$

[b]  $\lim_{x \rightarrow 2^+} \frac{x^3-16}{2-x}$

$= \infty$

$\frac{-8}{0^-}$  OR  $\frac{x^3-16 \rightarrow -8}{2-x \rightarrow 0^-}$

[c]  $\lim_{c \rightarrow 4} \frac{c-2}{c-4}$   $\frac{0}{0}$

$= \lim_{c \rightarrow 4} \frac{6-3(c-2)}{(c-4)(c-2)}$

$= \lim_{c \rightarrow 4} \frac{12-3c}{(c-4)(c-2)}$

$= \lim_{c \rightarrow 4} \frac{-3(c-4)}{(c-4)(c-2)}$

IGNORE -  
NOT  
REQUIRED

1 FOR FACTORING NUMERATOR

1 FOR CANCELLING

$= -\frac{3}{2}$

[d]  $\lim_{y \rightarrow -1} \frac{y^3-1}{3y^2+5y-12} = \frac{-2}{-14} = \frac{1}{7}$

deeeeeeeeeeeeeeeeeee  
SUBTRACT 1 POINT TOTAL  
IF YOU FORGOT ANY  
OF THESE  
deeeeeeeeeeeeeeeeeee