Math 1A (7:30am – 8:20am)
Quiz 3 Version A
Fri Jan 21, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION

YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS UNLESS SPECIFICALLY ASKED FOR

State the definition of "derivative (at a point)". **SCORE: ___ / 2 POINTS**

THE DERIVATIVE OF f AT a IS $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

OR $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

State the definition of "jump discontinuity". **SCORE: ___ / 2 POINTS**

f HAS A JUMP DISCONTINUITY AT a
IF $\lim\limits_{x \to a^+} f(x)$ AND $\lim\limits_{x \to a^-} f(x)$ BOTH EXIST BUT $\lim\limits_{x \to a^+} f(x) \neq \lim\limits_{x \to a^-} f(x)$

State the Intermediate Value Theorem. **SCORE: ___ / 2 POINTS**

IF f IS CONTINUOUS ON $[a, b]$ AND d IS BETWEEN $f(a)$ AND $f(b)$
THEN THERE EXISTS A $c \in (a, b)$ SUCH THAT $f(c) = d$

Let $f(x) = \begin{cases} cx^2 + 18, & \text{if } x < 2 \\ 6, & \text{if } x = 2 \\ 2 - cx^2, & \text{if } x > 2 \end{cases}$. **SCORE: ___ / 8 POINTS**

SUBTRACT 1 POINT IF YOU FOUND $\lim\limits_{x \to 2^-} (cx^2 + 18)$ ALSO

[a] If f is continuous from the right at $x = 2$, find the value of c. If there is no such value of c, write DNE and explain why.

$\lim\limits_{x \to 2^+} f(x) = \underbrace{\lim\limits_{x \to 2^+} (2 - cx^2)}_{1} = \underbrace{2 - 4c}_{\frac{1}{2}}$

$f(2) = 6$

IF $\underbrace{2 - 4c = 6}_{1}$, $\underbrace{c = -1}_{\frac{1}{2}}$

[b] If $c = -2$, is f continuous at $x = 2$?
If yes, show that all three conditions of continuity are satisfied. If no, determine the type of discontinuity.

$\lim\limits_{x \to 2^+} f(x) = \underbrace{\lim\limits_{x \to 2^+} (2 + 2x^2) = 10}_{1}$

$\lim\limits_{x \to 2^-} f(x) = \underbrace{\lim\limits_{x \to 2^-} (-2x^2 + 18) = 10}_{1}$

$\underbrace{\lim\limits_{x \to 2} f(x) = 10}_{1} \neq \underbrace{f(2)}_{1}$

$\underbrace{\text{REMOVABLE}}_{1}$

Find $\lim\limits_{x \to -\infty} \dfrac{\sqrt{2x^2+1}}{3x-5}$. **SCORE: ___ / 6 POINTS**

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2\left(2+\frac{1}{x^2}\right)}}{x\left(3-\frac{5}{x}\right)}$$

$$= \lim_{x \to -\infty} \frac{-\cancel{x}\sqrt{2+\frac{1}{x^2}}}{\cancel{x}\left(3-\frac{5}{x}\right)}$$

$2\frac{1}{2}$ $1\frac{1}{2}$

$$= -\frac{\sqrt{2+0}}{3-0}$$

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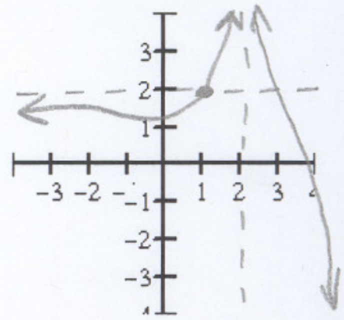
$$= -\frac{\sqrt{2}}{3}$$

1

Sketch the graph of a function that satisfies the following conditions, or write N/A if no such function exists. **SCORE: ___ / 2 POINTS**

$f(1)=2$, $\lim\limits_{x \to -\infty} f(x) = 2$, $\lim\limits_{x \to 2} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = -\infty$



Let $f(x) = \dfrac{4x}{x-1}$. **SCORE: ___ / 8 POINTS**

[a] Find $f'(3)$ using the definition of the derivative (at a point). **<u>DO NOT USE DIFFERENTIATION SHORTCUTS.</u>**

$$f'(3) = \lim_{x \to 3} \frac{\frac{4x}{x-1} - 6}{x-3}$$

3

$$= \lim_{x \to 3} \frac{4x-6(x-1)}{(x-3)(x-1)}$$

$$= \lim_{x \to 3} \frac{-2x+6}{(x-3)(x-1)}$$

$1\frac{1}{2}$

$$= \lim_{x \to 3} \frac{-2}{x-1}$$

1

$$= -1$$

1

OR

OR $f'(3) = \lim\limits_{h \to 0} \dfrac{\frac{4(3+h)}{(3+h)-1} - 6}{h}$

IGNORE

$$= \lim_{h \to 0} \frac{\frac{12+4h}{2+h} - 6}{h}$$

3

$$= \lim_{h \to 0} \frac{12+4h-6(2+h)}{h(2+h)}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(2+h)}$$

$1\frac{1}{2}$

$$= -1$$

1

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[b] Find the equation of the tangent line to $y = f(x)$ at $x = 3$.

$y - 6 = -(x-3)$ OR $y = -x+9$

$1\frac{1}{2}$