Math 1A (7:30am – 8:20am)
Quiz 3 Version B
Fri Jan 21, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION

YOU DO NOT NEED TO SHOW THE USE OF THE LIMIT LAWS UNLESS SPECIFICALLY ASKED FOR

State the definition of "derivative (at a point)". **SCORE: ___ / 2 POINTS**

SEE 7:30 VERSION A

State the definition of "removable discontinuity". **SCORE: ___ / 2 POINTS**

f HAS A REMOVABLE DISCONTINUITY AT a
IF $\lim_{x \to a} f(x)$ EXISTS BUT $f(a)$ DOES NOT EXIST
OR $\lim_{x \to a} f(x) \neq f(a)$

State the Intermediate Value Theorem. **SCORE: ___ / 2 POINTS**

SEE 7:30 VERSION A

Let $f(x) = \begin{cases} cx^2 + 18, & \text{if } x < 2 \\ 6, & \text{if } x = 2 \\ 2 - cx^2, & \text{if } x > 2 \end{cases}$. **SCORE: ___ / 8 POINTS**

SUBTRACT 1 POINT IF YOU FOUND $\lim_{x \to 2^+} (2 - cx^2)$ ALSO

[a] If f is continuous from the left at $x = 2$, find the value of c. If there is no such value of c, write DNE and explain why.

$\lim_{x \to 2^-} f(x) = \underbrace{\lim_{x \to 2^-} (cx^2 + 18)}_{1} = \underbrace{4c + 18}_{\frac{1}{2}}$

$f(2) = 6$

IF $\underbrace{4c + 18 = 6}_{1}$

$\underbrace{c = -3}_{\frac{1}{2}}$

[b] If $c = -4$, is f continuous at $x = 2$?
If yes, show that all three conditions of continuity are satisfied. If no, determine the type of discontinuity.

$\lim_{x \to 2^-} f(x) = \underbrace{\lim_{x \to 2^-} (-4x^2 + 18) = 2}_{1}$

$\lim_{x \to 2^+} f(x) = \underbrace{\lim_{x \to 2^+} (2 + 4x^2) = 18}_{1}$

$\lim_{x \to 2^-} f(x)$, $\lim_{x \to 2^+} f(x)$

$\underbrace{\text{BOTH EXIST}}_{1}$, BUT $\underbrace{\text{ARE NOT EQUAL}}_{1}$

$\underbrace{\text{JUMP}}_{1}$

Find $\lim\limits_{x \to -\infty} \dfrac{\sqrt{3x^2+1}}{2x-5}$. **SCORE: ___ / 6 POINTS**

$$= \lim_{x \to -\infty} \frac{\sqrt{x^2(3+\frac{1}{x^2})}}{x(2-\frac{5}{x})}$$

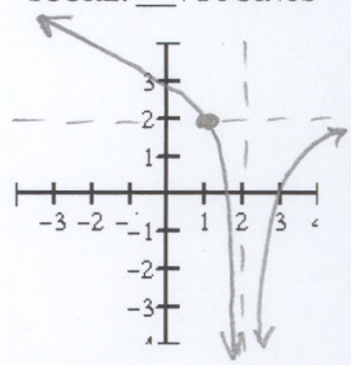
$$= \lim_{x \to -\infty} \frac{-\cancel{x}\sqrt{3+\frac{1}{x^2}}}{\cancel{x}(2-\frac{5}{x})} \quad 2\tfrac{1}{2} \quad 1\tfrac{1}{2}$$

$$= \frac{-\sqrt{3+0}}{2-0} \qquad 1 \text{ FOR CANCELLING}$$

$$= \frac{-\sqrt{3}}{2} \quad 1$$

Sketch the graph of a function that satisfies the following conditions, or write N/A if no such function exists. **SCORE: ___ / 2 POINTS**

$f(1) = 2$, $\lim\limits_{x \to \infty} f(x) = 2$, $\lim\limits_{x \to 2} f(x) = -\infty$ and $\lim\limits_{x \to -\infty} f(x) = \infty$



Let $f(x) = \sqrt{x^2 - 5}$. **SCORE: ___ / 8 POINTS**

[a] Find $f'(3)$ using the definition of the derivative (at a point). **<u>DO NOT USE DIFFERENTIATION SHORTCUTS.</u>**

$$f'(3) = \lim_{x \to 3} \frac{\sqrt{x^2-5}-2}{x-3} \quad 3$$

$$= \lim_{x \to 3} \frac{x^2-9}{(x-3)(\sqrt{x^2-5}+2)} \quad 1\tfrac{1}{2}$$

$$= \lim_{x \to 3} \frac{x+3}{\sqrt{x^2-5}+2} \quad 1$$

$$= \frac{6}{4}$$

$$= \frac{3}{2} \quad 1$$

OR $f'(3) = \lim\limits_{h \to 0} \dfrac{\sqrt{(3+h)^2-5}-2}{h}$

OR

$$= \lim_{h \to 0} \frac{\sqrt{h^2+6h+4}-2}{h} \quad 3$$

$$= \lim_{h \to 0} \frac{h^2+6h}{h(\sqrt{h^2+6h+4}+2)} \quad 1\tfrac{1}{2}$$

$$= \lim_{h \to 0} \frac{h+6}{\sqrt{h^2+6h+4}+2} \quad 1$$

$$= \frac{6}{4}$$

$$= \frac{3}{2} \quad 1$$

[b] Find the equation of the tangent line to $y = f(x)$ at $x = 3$.

$y - 2 = \frac{3}{2}(x-3)$ OR $y = \frac{3}{2}x - \frac{5}{2}$

$1\tfrac{1}{2}$