Math 1A (10:30am – 11:20am)
Quiz 4 Version B
Fri Feb 4, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION

State the definition of e given in section 3.1. **SCORE: ___ / 2 POINTS**

e IS THE NUMBER SUCH THAT $\underbrace{\lim_{h\to 0}}_{\frac{1}{2}} \underbrace{\frac{e^h-1}{h}}_{\frac{1}{2}} = \underbrace{1}_{\frac{1}{2}}$ $\frac{1}{2}$

Fill in the blanks. **SCORE: ___ / 3 POINTS**

$\frac{d}{dx}x^e = \underline{e x^{e-1}}$ 1 $\qquad \frac{d}{dx}e^3 = \underline{0}$ 1 $\qquad \frac{d}{dx}\pi^x = \underline{\pi^x \ln \pi}$ 1

Prove the derivative of $f(x) = \sec x$. **Show ALL steps (since you already know the final answer).** **SCORE: ___ / 4 POINTS**
Do not use the definition of the derivative.

$$f(x) = \frac{1}{\cos x}$$

$$f'(x) = \frac{(1)'(\cos x) - (1)(\cos x)'}{\cos^2 x} \quad \frac{1}{2}$$

$$= \frac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x} \quad 2$$

$$= \boxed{\frac{\sin x}{\cos^2 x}} \quad \frac{1}{2}$$

$$= \boxed{\frac{1}{\cos x}}\boxed{\frac{\sin x}{\cos x}} = \sec x \tan x$$

SUBTRACT $\frac{1}{2}$ POINT IF YOU FORGOT THIS

Prove the quotient rule using the definition of the derivative. **SCORE: ___ / 6 POINTS**
Show ALL steps (since you already know the final answer).

$$\lim_{h\to 0}\frac{(\frac{f}{g})(x+h) - (\frac{f}{g})(x)}{h} = \boxed{\lim_{h\to 0}\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}}\ 1 = \lim_{h\to 0}\boxed{\frac{f(x+h)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}}\ 1$$

$$= \lim_{h\to 0}\boxed{\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}}\ 1$$

$$= \lim_{h\to 0}\frac{1}{g(x+h)g(x)}\boxed{\left[\frac{f(x+h)-f(x)}{h}\cdot g(x) - f(x)\cdot\frac{g(x+h)-g(x)}{h}\right]}\ 1$$

$$= \boxed{\lim_{h\to 0}\frac{1}{g(x+h)g(x)}\left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\cdot\lim_{h\to 0}g(x) - \lim_{h\to 0}f(x)\cdot\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\right]}\ 1$$

$$= \frac{1}{[g(x)]^2}\left(f'(x)g(x) - f(x)g'(x)\right)$$

SUBTRACT $\frac{1}{2}$ POINT IF YOU FORGOT THIS LINE

+1 POINT FOR ALL THE "$\lim_{h\to 0}$"

Find the following derivatives. **Simplify and factor your final answers.** **SCORE: ___ / 8 POINTS**

[a] Find $\dfrac{d^2}{dx^2}\dfrac{x^2-3x-1}{\sqrt[3]{x}}$ $= \dfrac{d^2}{dx^2}\dfrac{x^2-3x-1}{x^{\frac{1}{3}}} = \dfrac{d^2}{dx^2}\left(x^{\frac{5}{3}} - 3x^{\frac{2}{3}} - x^{-\frac{1}{3}}\right)$

$= \dfrac{d}{dx}\left(\underbrace{\tfrac{5}{3}x^{\frac{2}{3}}}_{\frac12} \underbrace{- 2x^{-\frac{1}{3}}}_{\frac12} \underbrace{+ \tfrac{1}{3}x^{-\frac{4}{3}}}_{\frac12}\right)$

$= \underbrace{\tfrac{10}{9}x^{-\frac{1}{3}}}_{\frac12} \underbrace{+ \tfrac{2}{3}x^{-\frac{4}{3}}}_{\frac12} \underbrace{- \tfrac{4}{9}x^{-\frac{7}{3}}}_{\frac12}$

$= \underbrace{\tfrac{2}{9}x^{-\frac{7}{3}}}_{\frac12}\underbrace{(5x^2+3x-2)}_{\frac12}$

$= \tfrac{2}{9}x^{-\frac{7}{3}}\underbrace{(5x-2)(x+1)}_{\frac12 \text{ BONUS}}$

[b] Find the derivative of $f(x) = 5^x \csc x \cot x$.

$f'(x) = \underbrace{(5^x \ln 5)\csc x \cot x}_{1} + \underbrace{5^x(-\csc x \cot x)\cot x}_{1} + \underbrace{5^x \csc x(-\csc^2 x)}_{1}$

$= \underbrace{5^x \csc x}_{\frac12}\underbrace{((\ln 5)\cot x - \cot^2 x - \csc^2 x)}_{\frac12}$

Evaluate $\lim\limits_{x\to 0}\dfrac{1-x^2}{\cos x - 1}$. **SCORE: ___ / 3 POINTS**

The answer should be a number, ∞ or $-\infty$. **Write DNE only if the other possibilities do not apply.**

$\underbrace{\lim\limits_{x\to 0^+}\dfrac{1-x^2}{\cos x - 1} = -\infty}_{\frac{1}{0^-}}$ 1

$\underbrace{\lim\limits_{x\to 0^-}\dfrac{1-x^2}{\cos x - 1} = -\infty}_{\frac{1}{0^-}}$ 1

$\lim\limits_{x\to 0}\dfrac{1-x^2}{\cos x - 1} = -\infty$ 1

Find the equation of the normal line to $y = \dfrac{2x^3-3x-5}{3x^2-1}$ at the point where $x = 1$. **SCORE: ___ / 4 POINTS**

$\dfrac{dy}{dx} = \dfrac{\underbrace{(6x^2-3)(3x^2-1)}_{1} - \underbrace{(2x^3-3x-5)(6x)}_{1}}{\underbrace{(3x^2-1)^2}_{\frac12}}$

$\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{3(2)-(-6)(6)}{2^2} = \underbrace{\dfrac{21}{2}}_{\frac12}$

$m = -\dfrac{2}{21}$

$\underbrace{y-(-3) = \underbrace{-\dfrac{2}{21}}_{\frac12}(x-1)}_{\frac12}$