Math 1A (7:30am – 8:20am)
Quiz 4 Version A
Fri Feb 4, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION

State the definition of e given in section 3.1. **SCORE: ___ / 2 POINTS**

$\underbrace{\text{e IS THE NUMBER SUCH THAT}}_{\frac{1}{2}}$ $\underbrace{\lim_{h \to 0}}_{\frac{1}{2}} \underbrace{\frac{e^h - 1}{h}}_{\frac{1}{2}} \underbrace{= 1}_{\frac{1}{2}}$

Fill in the blanks. **SCORE: ___ / 3 POINTS**

$\frac{d}{dx} e^3 = 0$ 1 $\qquad \frac{d}{dx} \pi^x = \pi^x \ln \pi$ 1 $\qquad \frac{d}{dx} x^e = e x^{e-1}$ 1

Prove the derivative of $f(x) = \csc x$. **Show ALL steps (since you already know the final answer).** **SCORE: ___ / 4 POINTS**
Do not use the definition of the derivative.

$f(x) = \frac{1}{\sin x}$

$f'(x) = \frac{(1)'(\sin x) - (1)(\sin x)'}{\underbrace{\sin^2 x}_{\frac{1}{2}}}$

$= \frac{\underbrace{0 \cdot \sin x - 1 \cdot \cos x}_{2}}{\sin^2 x}$

$= \boxed{\frac{-\cos x}{\sin^2 x}}$ $\frac{1}{2}$

SUBTRACT $\frac{1}{2}$ POINT IF YOU FORGOT THIS

$= -\underbrace{\boxed{\frac{1}{\sin x}}}_{\frac{1}{2}} \cdot \underbrace{\boxed{\frac{\cos x}{\sin x}}}_{\frac{1}{2}} = -\csc x \cot x$

Prove the quotient rule using the definition of the derivative. **SCORE: ___ / 6 POINTS**
Show ALL steps (since you already know the final answer).

$\lim_{h \to 0} \frac{\left(\frac{f}{g}\right)(x+h) - \left(\frac{f}{g}\right)(x)}{h}$

$= \boxed{\lim_{h \to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}}$ 1

$= \lim_{h \to 0} \boxed{\frac{f(x+h)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}}$ 1

$= \lim_{h \to 0} \boxed{\frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+h)}{h\,g(x+h)g(x)}}$ 1

$= \lim_{h \to 0} \frac{1}{g(x+h)g(x)} \left[\boxed{\frac{f(x+h) - f(x)}{h} \cdot g(x) - f(x) \cdot \frac{g(x+h) - g(x)}{h}} \right]$ 1 1

$= \boxed{\lim_{h \to 0} \frac{1}{g(x+h)g(x)} \cdot \left[\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \lim_{h \to 0} g(x) - \lim_{h \to 0} f(x) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \right]}$ 1

$= \frac{1}{[g(x)]^2} \left(f'(x)g(x) - f(x)g'(x) \right)$

SUBTRACT $\frac{1}{2}$ POINT IF YOU FORGOT THIS LINE

Find the following derivatives. **Simplify and factor your final answers.** **SCORE: ___ / 8 POINTS**

[a] Find $\dfrac{d^2}{dx^2}\dfrac{2x^2-x-1}{\sqrt[4]{x}}$

$$= \frac{d^2}{dx^2}\,\frac{2x^2-x-1}{x^{\frac14}} = \frac{d^2}{dx^2}\left(2x^{\frac74}-x^{\frac34}-x^{-\frac14}\right)$$

$$= \frac{d}{dx}\left(\underbrace{\tfrac72 x^{\frac34}}_{\frac12}-\underbrace{\tfrac34 x^{-\frac14}}_{\frac12}+\underbrace{\tfrac14 x^{-\frac54}}_{\frac12}\right)$$

$$= \underbrace{\tfrac{21}{8}x^{-\frac14}}_{\frac12}+\underbrace{\tfrac{3}{16}x^{-\frac54}}_{\frac12}-\underbrace{\tfrac{5}{16}x^{-\frac94}}_{\frac12}$$

$$= \underbrace{\tfrac{1}{16}x^{-\frac94}}_{\frac12}\underbrace{\left(42x^2+3x-5\right)}_{\frac12}$$

[b] Find the derivative of $f(x)=7^x\sec x\tan x$.

$$f'(x)=\underbrace{(7^x\ln 7)\sec x\tan x}_{1}+\underbrace{7^x(\sec x\tan x)\tan x}_{1}$$

$$+\underbrace{7^x\sec x(\sec^2 x)}_{1}$$

$$=\underbrace{7^x\sec x}_{\frac12}\underbrace{\left((\ln 7)\tan x+\tan^2 x+\sec^2 x\right)}_{\frac12}$$

Evaluate $\displaystyle\lim_{x\to 0}\frac{x^2-1}{\cos x-1}$. **SCORE: ___ / 3 POINTS**

The answer should be a number, ∞ or $-\infty$. **Write DNE only if the other possibilities do not apply.**

$$\underbrace{\lim_{x\to 0^+}\frac{x^2-1}{\cos x-1}=\infty}_{\frac{-1}{0^-}}\quad 1$$

$$\underbrace{\lim_{x\to 0^-}\frac{x^2-1}{\cos x-1}=\infty}_{\frac{-1}{0^-}}\quad 1$$

$$\lim_{x\to 0}\frac{x^2-1}{\cos x-1}=\infty\quad 1$$

Find the equation of the normal line to $y=\dfrac{2x^4-3x^2-5}{2x^3-1}$ at the point where $x=1$. **SCORE: ___ / 4 POINTS**

$$\frac{dy}{dx}=\frac{\overset{1}{(8x^3-6x)(2x^3-1)}-\overset{1}{(2x^4-3x^2-5)(6x^2)}}{\underbrace{(2x^3-1)^2}_{\frac12}}$$

$$\left.\frac{dy}{dx}\right|_{x=1}=\frac{2(1)-(-6)6}{1^2}=\underbrace{38}_{\frac12}$$

$$m=-\frac{1}{38}$$

$$\underbrace{y-(-6)=\underbrace{-\tfrac{1}{38}}_{\frac12}(x-1)}_{\frac12}$$