Math 1A (10:30am – 11:20am)
Quiz 5 Version A
Fri Feb 11, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION

Can the Intermediate Value Theorem be used to prove that $f(x) = 1 - 4x^{-2}$ passes through the x – axis somewhere in the interval $[-1, 4]$? Why or why not ? **SCORE: ___ / 3 POINTS**

NO, f IS NOT CONTINUOUS AT $x = 0 \in [-1, 4]$ $\frac{1}{2}$
$1\frac{1}{2}$ SINCE f(0) DNE, SO THE IVT DOES NOT APPLY.
1

The value of $f(x)$ at various inputs is given below. **SCORE: ___ / 4 POINTS**

x	-1	0	1	2	3	4	5
$f(x)$	3	2	4	1	-2	-3	-1
$f'(x)$	-2	0	-5	-4	-2	2	4

If $g(x) = x^4 f(2x^2)$, find $g'(-1)$.

$$g'(x) = \underbrace{4x^3 f(2x^2)}_{1} + \underbrace{x^4 f'(2x^2)(4x)}_{1}$$

$$g'(-1) = 4(-1)^3 f(2(-1)^2) + (-1)^4 f'(2(-1)^2)(4(-1))$$

$$= -4f(2) - 4f'(2)$$

$$= \underbrace{-4}_{\frac{1}{4}}\underbrace{(1)}_{\frac{1}{2}} \underbrace{- 4}_{\frac{1}{4}}\underbrace{(-4)}_{\frac{1}{2}}$$

$$= \underbrace{12}_{\frac{1}{2}}$$

Use the tangent line to $f(x) = \tan^{-1} x$ at $x = -1$ to approximate $\tan^{-1}(-1.2)$. **SCORE: ___ / 4 POINTS**
Your final answer may involve e, π and/or radicals.

$$f(x) \approx L(x) = f(-1) + f'(-1)(x - {-1})$$

$$= \tan^{-1}(-1) + \frac{1}{1+(-1)^2}(x+1)$$

$$= -\frac{\pi}{4} + \frac{1}{2}(x+1)$$

$$\tan^{-1}(-1.2) = f(-1.2)$$

$$\approx L(-1.2)$$

$$= -\frac{\pi}{4} + \frac{1}{2}(-1.2+1) = \overbrace{-\frac{\pi}{4}}^{1} + \overbrace{\frac{1}{2}}^{1}\overbrace{\left(-\frac{1}{5}\right)}^{1} = \overbrace{-\frac{\pi}{4} - \frac{1}{10}}^{1}$$

Find the slope of the tangent line to the curve $(6+x^3y^2)^4 = 16(9x^2 - y^3)$ at the point $(-1, 2)$. **SCORE: ___ / 5 POINTS**

$$\underbrace{4(6+x^3y^2)^3}_{}(\underbrace{3x^2y^2}_{\frac{1}{2}} + \underbrace{x^3(2y)\frac{dy}{dx}}_{1}) = 16(\underbrace{18x}_{\frac{1}{2}} - \underbrace{3y^2\frac{dy}{dx}}_{1})$$

$$4(6+(-1)^3(2)^2)^3(3(-1)^2(2)^2 + (-1)^3 2(2)\frac{dy}{dx}\Big|_{(-1,2)}) = 16(18(-1) - 3(2)^2\frac{dy}{dx}\Big|_{(-1,2)})$$

$$\cancel{8}\ \cancel{32}(12 - 4\frac{dy}{dx}\Big|_{(-1,2)}) = \cancel{16}(\cancel{-18}^{-9} - \cancel{12}^{-6}\frac{dy}{dx}\Big|_{(-1,2)})$$

$$21 = -2\frac{dy}{dx}\Big|_{(-1,2)} \qquad \frac{dy}{dx}\Big|_{(-1,2)} = \underbrace{-\frac{21}{2}}_{1}$$

Find the following derivatives. Simplify and factor your answers. **SCORE: ___ / 9 POINTS**

[a] If $f(x) = \dfrac{\ln x}{\ln(\ln x)}$, find $f'(x)$. **NOTE: $f(x)$ can NOT be simplified.**

$$f'(x) =_{1} \frac{\underbrace{\frac{1}{x}\ln(\ln x) - (\ln x)\frac{1}{\ln x}\cdot\frac{1}{x}}_{2}}{\underbrace{(\ln(\ln x))^2}_{1}}$$

$$= \boxed{\frac{\ln(\ln x) - 1}{x(\ln(\ln x))^2}} \quad 1$$

[b] Find $\dfrac{d}{dx}\sin^{-1}\sqrt{1-x^8}$.

$$= \underbrace{\frac{1}{\sqrt{1-\sqrt{1-x^8}^{\,2}}}}_{1} \cdot \underbrace{\frac{1}{2\sqrt{1-x^8}}}_{1} \cdot \underbrace{-8x^7}_{1}$$

$$= \frac{1}{x^4}\cdot\frac{1}{2\sqrt{1-x^8}}\cdot -8x^7$$

$$= \boxed{\frac{-4x^3}{\sqrt{1-x^8}}} \quad 1$$

Prove the derivative of $f(x) = \arccos x$. **Show ALL steps (since you already know the final answer).** **SCORE: ___ / 5 POINTS**

LET $\underbrace{y = \arccos x}_{\frac{1}{2}}$

$\frac{1}{2}$ $\underbrace{\cos y = x}$ AND $\underbrace{0 \le y \le \pi \Rightarrow \sin y \ge 0}_{1} \Rightarrow \sin y = \sqrt{1-\cos^2 y}$

1 $\underbrace{-\sin y\frac{dy}{dx} = 1}$

$$\underbrace{\frac{dy}{dx} = -\frac{1}{\sin y}}_{\frac{1}{2}} = \underbrace{-\frac{1}{\sqrt{1-\cos^2 y}}}_{1} = \underbrace{-\frac{1}{\sqrt{1-x^2}}}_{\frac{1}{2}}$$