Math 1A (7:30am – 8:20am)
Quiz 5 Version B
Fri Feb 11, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION

Can the Intermediate Value Theorem be used to prove that $f(x) = 1 - 4x^{-2}$ passes through the $x-$axis somewhere in the interval $[-1, 4]$? Why or why not ? **SCORE: ___ / 3 POINTS**

NO, $1\frac{1}{2}$ f IS DISCONTINUOUS AT $x = 0 \in [-1, 4]$ $\frac{1}{2}$
SINCE f(0) DNE, SO THE IVT DOES NOT APPLY, 1

The value of $f(x)$ at various inputs is given below. **SCORE: ___ / 4 POINTS**

x	-1	0	1	2	3	4	5
$f(x)$	3	2	4	1	-2	-3	-1
$f'(x)$	-2	0	-5	-4	-2	2	4

If $g(x) = x^4 f(x^2)$, find $g'(-1)$.

$$g'(x) = \underbrace{4x^3 f(x^2)}_{1} + \underbrace{x^4 f'(x^2)(2x)}_{1}$$

$$g'(-1) = 4(-1)^3 f((-1)^2) + (-1)^4 f'((-1)^2)(2(-1))$$

$$= -4f(1) - 2f'(1)$$

$$= \underbrace{-4}_{\frac{1}{4}}\underbrace{(4)}_{\frac{1}{2}}\underbrace{-2}_{\frac{1}{4}}\underbrace{(-5)}_{\frac{1}{2}}$$

$$= \underbrace{-6}_{\frac{1}{2}}$$

Use the tangent line to $f(x) = \tan^{-1} x$ at $x = 1$ to approximate $\tan^{-1} 0.8$. **SCORE: ___ / 4 POINTS**
Your final answer may involve e, π **and/or radicals.**

$$f(x) \approx L(x) = f(1) + f'(1)(x-1)$$

$$= \tan^{-1} 1 + \frac{1}{1+1^2}(x-1)$$

$$= \frac{\pi}{4} + \frac{1}{2}(x-1)$$

$$\tan^{-1} 0.8 = f(0.8)$$

$$\approx L(0.8)$$

$$= \overbrace{\frac{\pi}{4}}^{1} + \frac{1}{2}(0.8-1) = \overbrace{\frac{\pi}{4}}^{1} + \overbrace{\frac{1}{2}\left(-\frac{1}{5}\right)}^{1} = \overbrace{\frac{\pi}{4} - \frac{1}{10}}^{1}$$

Find the slope of the tangent line to the curve $(6+x^3y^2)^4 = 16(y^3-7x^2)$ at the point $(-1, 2)$. **SCORE: ___ / 5 POINTS**

$$4(6+x^3y^2)^3(3x^2y^2 + x^3(2y)\frac{dy}{dx}) = 16(3y^2\frac{dy}{dx} - 14x)$$

1 1 ½ 1 1 ½

$$4(6+(-1)^3 2^2)^3(3(-1)^2 2^2 + (-1)^3 2(2)\frac{dy}{dx}\Big|_{(-1,2)}) = 16(3(2)^2\frac{dy}{dx}\Big|_{(-1,2)} - 14(-1))$$

$$\cancel{2}\,\cancel{32}(12 - 4\frac{dy}{dx}\Big|_{(-1,2)}) = \cancel{16}(\overset{6}{\cancel{12}}\frac{dy}{dx}\Big|_{(-1,2)} + \overset{7}{\cancel{14}})$$

$$5 = 10\frac{dy}{dx}\Big|_{(-1,2)} \quad \frac{dy}{dx}\Big|_{(-1,2)} = \frac{1}{2}$$

1

Find the following derivatives. Simplify and factor your answers. **SCORE: ___ / 9 POINTS**

[a] If $f(x) = \dfrac{\ln(\ln x)}{\ln x}$, find $f'(x)$. **NOTE:** $f(x)$ **can NOT be simplified.**

$$f'(x) = \frac{\cancel{\frac{1}{\ln x}} \cdot \frac{1}{x} \cdot \cancel{\ln x} - \ln(\ln x)\cdot\frac{1}{x}}{(\ln x)^2}$$

2 1 1

$$= \boxed{\frac{1-\ln(\ln x)}{x(\ln x)^2}}$$

1

[b] Find $\dfrac{d}{dx}\sin^{-1}\sqrt{1-x^4}$.

$$= \frac{1}{\sqrt{1-\sqrt{1-x^4}^{\,2}}} \cdot \frac{1}{2\sqrt{1-x^4}} \cdot -4x^3$$

1 1 1

$$= \frac{1}{x^2} \cdot \frac{1}{2\sqrt{1-x^4}} \cdot -4x^3$$

$$= \boxed{\frac{-2x}{\sqrt{1-x^4}}}$$

1

Prove the derivative of $f(x) = \arccos x$. **Show ALL steps (since you already know the final answer).** **SCORE: ___ / 5 POINTS**

LET $y = \arccos x$, ½

½ $\cos y = x$, AND $0 \le y \le \pi \Rightarrow \sin y \ge 0 \Rightarrow \sin y = \sqrt{1-\cos^2 y}$

1

1 $-\sin y \frac{dy}{dx} = 1$

$$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$$

½ 1 ½