

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED**SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION**

State the definition of "concave up".

SCORE: ___ / 2 POINTS

f IS CONCAVE UP ON (a, b)
 IF f' IS INCREASING ON (a, b)

State the Mean Value Theorem.

SCORE: ___ / 2 POINTS

IF f IS CONTINUOUS ON $[a, b]$ AND DIFFERENTIABLE ON (a, b)
 THEN THERE EXISTS A VALUE $c \in (a, b)$ SUCH THAT $f'(c) = \frac{f(b) - f(a)}{b - a}$

Let $f(x) = 2 - 15x^4 - 3x^5$.

SCORE: ___ / 10 POINTS

[a] Find all critical numbers of f .

$$f'(x) = -60x^3 - 15x^4$$

$$= -15x^3(4 + x) = 0 \quad @ \quad x = 0, -4$$

[b] For each critical number, determine what the Second Derivative Test tells you about that critical number.

$$f''(x) = -180x^2 - 60x^3$$

$$= -60x^2(3 + x)$$

$$\frac{1}{2} f''(0) = 0 \rightarrow \text{NO CONCLUSION}$$

$$\frac{1}{2} f''(-4) > 0 \rightarrow \text{LOCAL MIN @ } x = -4$$

[c] Find the inflection points of f .

$$f'' = 0 \quad @ \quad x = 0, -3$$

f''	+	-	-
	-3	0	
$-60x^2$	-	-	-
$3+x$	-	+	+

$$\text{I.P. @ } x = -3$$

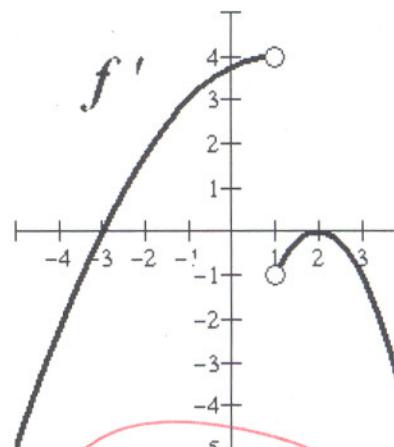
NO POINTS IF YOU
 SAID BOTH $x = 0, -3$

The graph of the derivative f' of a continuous function f is shown on the right.

SCORE: ___ / 10 POINTS

- [a] Write "OK" if you understand that the graph shows f' and NOT f , but that all the questions below are about f and NOT f' , and that you must explain your answers to get any credit.

OK



- [b] Is f increasing or decreasing on the interval $[-5, -3]$? Explain your answer(s) very briefly.

$f' < 0 \rightarrow f$ DECREASING

- [c] Find the x -coordinates of all critical numbers of f . Explain your answer(s) very briefly.

$f' = 0 @ x = -3, 2$

f' DNE @ $x = 1$

1 POINT EACH

- [d] Find the x -coordinates of all local maxima of f . Explain your answer(s) very briefly.

f' CHANGES FROM POSITIVE TO NEGATIVE @ $x = 1$

- [e] Is f concave up or down on the interval $[-3, 0]$? Explain your answer(s) very briefly.

f' INCREASING $\rightarrow f$ CONCAVE UP

- [f] Find the x -coordinates of all inflection points of f . Explain your answer(s) very briefly.

f' CHANGES FROM INCREASING TO DECREASING @ $x = 2$

Does Rolle's Theorem apply to the function $f(x) = \sqrt[3]{x} - \frac{x}{9}$ on the interval $[-27, 27]$?

SCORE: ___ / 3 POINTS

If yes, find the value of c guaranteed by Rolle's Theorem. If no, explain why not.

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{9}$ DNE @ $x = 0$

f IS NOT DIFFERENTIABLE ON $[-27, 27]$

SO, ROLLE'S THEOREM DOES NOT APPLY

Sketch the graph of a continuous function that satisfies all the given conditions.

SCORE: ___ / 3 POINTS

$f'(x) < 0$ if $x < 2$,

$f'(x) > 0$ if $x > 2$,

$f''(x) > 0$ if $|x| > 2$,

$f''(x) < 0$ if $|x| < 2$

