

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED**SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION**

State the definition of "concave down".

SCORE: ____ / 2 POINTS

f IS CONCAVE DOWN ON (a, b)
 IF f' IS DECREASING ON (a, b)

State the Mean Value Theorem.

SCORE: ____ / 2 POINTS

IF f IS CONTINUOUS ON $[a, b]$ AND DIFFERENTIABLE ON (a, b)
 THEN THERE EXISTS A VALUE $c \in (a, b)$ SUCH THAT $f'(c) = \frac{f(b) - f(a)}{b - a}$

Let $f(x) = 9 + 10x^4 - 2x^5$.

SCORE: ____ / 10 POINTS

[a] Find all critical numbers of f .

$$f'(x) = 40x^3 - 10x^4$$

$$= 10x^3(4 - x) = 0 \text{ @ } x = 0, 4$$

[b] For each critical number, determine what the Second Derivative Test tells you about that critical number.

$$f''(x) = 120x^2 - 40x^3$$

$$= 40x^2(3 - x)$$

$$\frac{1}{2} f''(0) = 0 \rightarrow \text{NO CONCLUSION}$$

$$\frac{1}{2} f''(4) < 0 \rightarrow \text{LOCAL MAX @ } x = 4$$

[c] Find the inflection points of f .

$$f'' = 0 \text{ @ } x = 0, 3$$

f''	+	+	-
	0		3
$40x^2$	+	+	+
$3 - x$	+	+	-

$$\text{I.P. @ } x = 3$$

NO POINTS IF YOU
SAID BOTH $x = 0, 3$

The graph of the derivative f' of a continuous function f is shown on the right.

SCORE: ___ / 10 POINTS

- [a] Write "OK" if you understand that the graph shows f' and NOT f , but that all the questions below are about f and NOT f' , and that you must explain your answers to get any credit.

OK

- [b] Is f increasing or decreasing on the interval $[0, 2]$? Explain your answer(s) very briefly.

$f' < 0 \rightarrow f$ DECREASING.

1 POINT EACH

- [c] Find the x -coordinates of all critical numbers of f . Explain your answer(s) very briefly.

$f' = 0$ @ $x = -2, 3$ f' DNE @ $x = -1$

- [d] Find the x -coordinates of all local maxima of f . Explain your answer(s) very briefly.

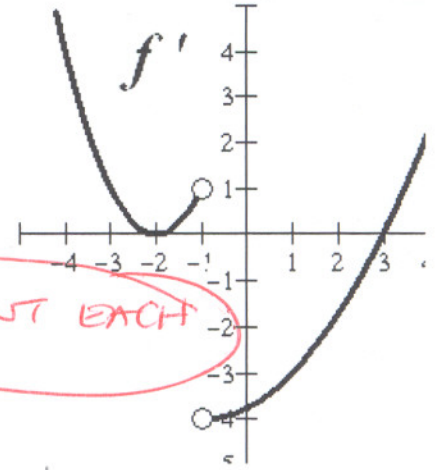
f' CHANGES FROM POSITIVE TO NEGATIVE @ $x = -1$

- [e] Is f concave up or down on the interval $[-4, -2]$? Explain your answer(s) very briefly.

f' DECREASING $\rightarrow f$ CONCAVE DOWN

- [f] Find the x -coordinates of all inflection points of f . Explain your answer(s) very briefly.

f' CHANGES FROM DECREASING TO INCREASING @ $x = -2$



Does Rolle's Theorem apply to the function $f(x) = \sqrt[3]{x} - \frac{x}{4}$ on the interval $[-8, 8]$?

SCORE: ___ / 3 POINTS

If yes, find the value of c guaranteed by Rolle's Theorem. If no, explain why not.

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{4}$ DNE @ $x = 0$ $\frac{1}{2}$

f IS NOT DIFFERENTIABLE ON $[-8, 8]$

SO ROLLE'S THEOREM DOES NOT APPLY

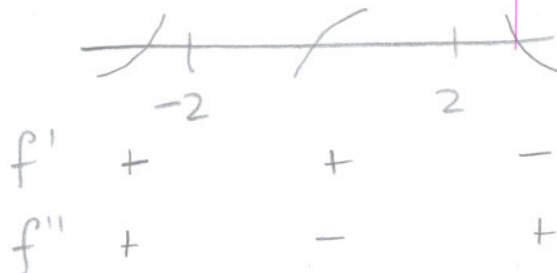
Sketch the graph of a continuous function that satisfies all the given conditions.

$$f'(x) > 0 \text{ if } x < 2,$$

$$f'(x) < 0 \text{ if } x > 2,$$

$$f''(x) > 0 \text{ if } |x| > 2,$$

$$f''(x) < 0 \text{ if } |x| < 2$$



SCORE: ___ / 3 POINTS

