Math 1A (10:30am – 11:20am)
Quiz 8 Version B
Fri Mar 11, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

SHOW PROPER ALGEBRAIC WORK AND USE PROPER NOTATION

Find the following limits. **SCORE: ___ / 18 POINTS**
Your final answer should be a number, ∞ or $-\infty$. **Write DNE only if none of the other answers apply.**

[a] $\lim_{x\to 0^+} \dfrac{e^{2x}-2x-1}{2-x^2-2\cos x}$ $\quad \frac{1-0-1}{2-0-2} \sim \frac{0}{0}$

$= \lim_{x\to 0^+} \dfrac{2e^{2x}-2}{-2x+2\sin x}$ $\quad \frac{2-2}{0+0} \sim \frac{0}{0}$

$= \lim_{x\to 0^+} \dfrac{4e^{2x}}{-2+2\cos x}$ $\quad \frac{4}{0}$

$= -\infty$

[b] $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$ L'HÔPITAL'S RULE GOES INTO CIRCLE (SEE 4.4 #71)

$= \lim_{x\to\infty} \dfrac{x}{\sqrt{x^2}\sqrt{1+\frac{1}{x^2}}}$

$= \lim_{x\to\infty} \dfrac{x}{x\sqrt{1+\frac{1}{x^2}}}$

$= \dfrac{1}{\sqrt{1+0}}$

$= 1$

[c] $\lim_{x\to 0^+} (1-\sin 4x)^{\cot x}$ $\quad (1-0)^{\frac{1}{0}} \sim 1^{\infty}$

$\lim_{x\to 0^+} \ln(1-\sin 4x)^{\cot x}$

$= \lim_{x\to 0^+} \cot x \ln(1-\sin 4x)$ $\quad \infty \cdot 0$

$= \lim_{x\to 0^+} \dfrac{\ln(1-\sin 4x)}{\tan x}$ $\quad \frac{0}{0}$

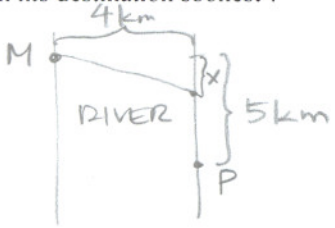
$= \lim_{x\to 0^+} \dfrac{\frac{1}{1-\sin 4x}\cdot -4\cos 4x}{\sec^2 x}$

$= \dfrac{\frac{1}{1-0}\cdot -4\cdot 1}{1^2} = -4$

$\lim_{x\to 0^+} (1-\sin 4x)^{\cot x} = e^{-4}$

A man launches his boat from the left bank of a straight river, 4 km wide, and wants to reach a point 5 km downstream on the right bank. If he can row 4 km per hour and walk 5 km per hour, where should he land his boat on the right bank to reach his destination soonest ?

SCORE: ___ / 12 POINTS



MINIMIZE t = TIME TO REACH DESTINATION

BY CHANGING x = DISTANCE DOWNSTREAM HE LANDS ON RIGHT BANK

$$t = \frac{\sqrt{x^2+16}}{4} + \frac{5-x}{5} \qquad x \in [0,5]$$

$$t' = \frac{1}{4}\frac{1}{2\sqrt{x^2+16}}\cdot 2x - \frac{1}{5}$$

$$= \frac{x}{4\sqrt{x^2+16}} - \frac{1}{5} = 0$$

$$\frac{x}{4\sqrt{x^2+16}} = \frac{1}{5}$$

$$5x = 4\sqrt{x^2+16}$$

$$25x^2 = 16(x^2+16)$$

$$25x^2 = 16x^2+256$$

$$9x^2 = 256$$

$$x^2 = \frac{256}{9}$$

$$x = \pm\frac{16}{3} \notin [0,5]$$

$$t(0) = \frac{\sqrt{16}}{4} + \frac{5}{5} = 2$$

$$t(5) = \frac{\sqrt{41}}{4} + \frac{0}{5} < \frac{\sqrt{49}}{4} = \frac{7}{4}$$

HE SHOULD ROW DIRECTLY TO HIS DESTINATION AND NOT WALK AT ALL