Math 49B (8:30am – 9:20am)
Midterm 1 Version A
Fri Jan 28, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT

Simplify $\cos 78^\circ \cos 42^\circ - \sin 78^\circ \sin 42^\circ$. **SCORE: ___ / 6 POINTS**

$= \cos(78^\circ + 42^\circ)$

$= \cos 120^\circ$

$= -\frac{1}{2}$

Fill in the blanks. Write DNE if the expression has no value. **SCORE: ___ / 24 POINTS**

[a] $\sin^{-1}(-1) =$ $-\frac{\pi}{2}$

[b] $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$

[c] $\tan^{-1}(-\sqrt{3}) =$ $-\frac{\pi}{3}$

[d] $\sin(\arcsin \pi) =$ DNE

[e] $\cos^{-1}\left(\cos \frac{2\pi}{3}\right) =$ $\frac{2\pi}{3}$

[f] $\tan^{-1}\left(\tan \frac{5\pi}{6}\right) =$ $-\frac{\pi}{6}$

[g] The period of $y = \sec x$ is 2π.

[h] The range of $y = \csc x$ is $(-\infty, -1] \cup [1, \infty)$

[i] The period of $y = \cot x$ is π.

[j] The domain of $y = \sec x$ is $\{x \neq \frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\}$.

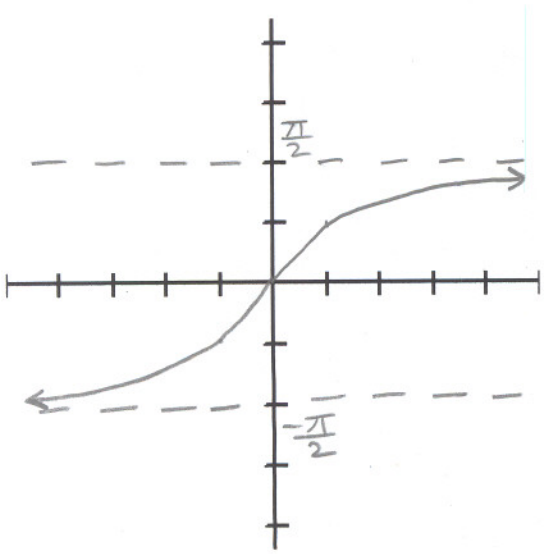
[k] The domain of $y = \tan^{-1} x$ is $(-\infty, \infty)$.

[l] The range of $y = \cos^{-1} x$ is $[0, \pi]$.

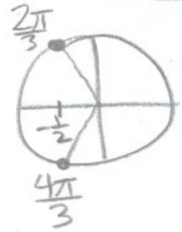
[m] The **equations** of the asymptotes of $y = \cot x$ are $x = n\pi,\ n \in \mathbb{Z}$.

Graph $y = \tan^{-1} x$. Label the important $x-$ and/or $y-$co-ordinates shown in class. **SCORE: ___ / 9 POINTS**

Draw your graph on the axes included below.



Find all solutions of the equation $1+2\cos\frac{x}{2}=0$ in the interval $[0, 2\pi)$. **SCORE: ___ / 10 POINTS**



$$\cos\frac{x}{2} = -\frac{1}{2}$$

☆ SEE ALSO VERSION B (PREFERRED)

$$\frac{x}{2} = \frac{2\pi}{3} + 2n\pi \text{ OR } \frac{4\pi}{3} + 2n\pi$$

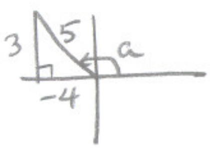
$$x = \frac{4\pi}{3} + 4n\pi \text{ OR } \frac{8\pi}{3} + 4n\pi$$

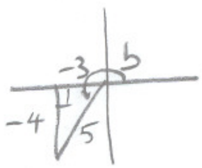
IF $n=-1$, $x = \frac{4\pi}{3} - 4\pi < 0$ OR $x = \frac{8\pi}{3} - 4\pi < 0$

$n=0$, $x = \frac{4\pi}{3} \in [0, 2\pi)$ OR $x = \frac{8\pi}{3} > 2\pi$

$n=1$, $x = \frac{4\pi}{3} + 4\pi > 2\pi$ SO $x = \frac{4\pi}{3}$

If $\sin a = \frac{3}{5}$ and $\frac{\pi}{2} \le a \le \pi$, and $\sin b = -\frac{4}{5}$ and $\pi \le b \le \frac{3\pi}{2}$, find $\tan(a+b)$. **SCORE: ___ / 15 POINTS**





$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$= \frac{-\frac{3}{4} + \frac{4}{3}}{1 - (-\frac{3}{4})(\frac{4}{3})} = \frac{\frac{7}{12}}{2} = \frac{7}{24}$$

Simplify the expression $\cot\left(\frac{\pi}{2} - x\right)\csc(-x)$. **SCORE: ___ / 8 POINTS**

$$= \tan x\,(-\csc x)$$

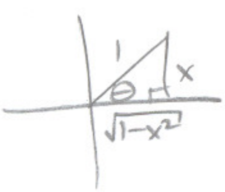
$$= \frac{\sin x}{\cos x} \cdot -\frac{1}{\sin x}$$

$$= -\frac{1}{\cos x} = -\sec x$$

If $x > 0$, write $\sin(2\arcsin x)$ as an algebraic expression (ie. an expression without trigonometric functions). **SCORE: ___ / 15 POINTS**

LET $\theta = \arcsin x$

$\sin\theta = x$ AND $\theta \in Q_1$



$$\sin(2\arcsin x) = 2\sin\theta\cos\theta$$

$$= 2x\sqrt{1-x^2}$$

Find all solutions of the equation $\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right)=-1$. **SCORE: ___ / 18 POINTS**

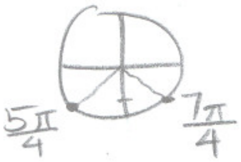
$$\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}+\sin x\cos\frac{\pi}{4}-\cos x\sin\frac{\pi}{4}=-1$$

$$2\sin x\cos\frac{\pi}{4}=-1$$

$$(2\sin x)\frac{\sqrt{2}}{2}=-1$$

$$\sqrt{2}\sin x=-1$$

$$\sin x=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$



$$x=\frac{5\pi}{4}+2n\pi \text{ OR } \frac{7\pi}{4}+2n\pi$$

Use the power-reducing formulae to rewrite $\sin^4 x$ in terms of first powers of cosines. **SCORE: ___ / 12 POINTS**

$$\sin^4 x=(\sin^2 x)^2$$

$$=\left(\frac{1-\cos 2x}{2}\right)^2$$

$$=\frac{1-2\cos 2x+\cos^2 2x}{4}$$

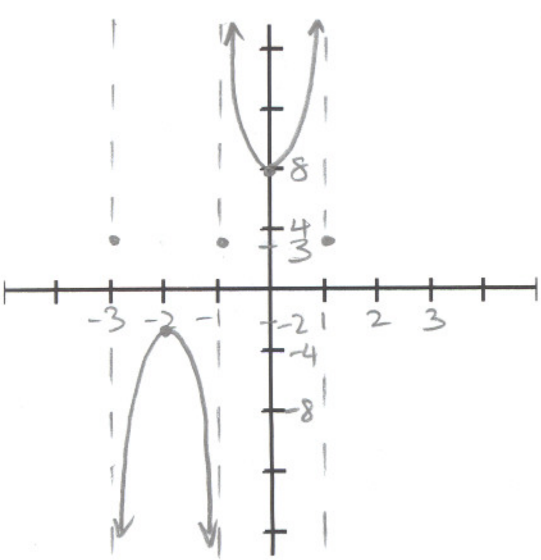
$$=\frac{1-2\cos 2x+\frac{1+\cos 4x}{2}}{4}$$

$$=\frac{2-4\cos 2x+1+\cos 4x}{8}=\frac{3-4\cos 2x+\cos 4x}{8}$$

Graph one period of the function $y=-5\csc\left(\frac{\pi x}{2}+\frac{3\pi}{2}\right)+3$. **SCORE: ___ / 18 POINTS**

Label the important $x-$ and/or $y-$ co-ordinates shown in class.

Draw your graph on the axes included below. Label your axes so the entire graph is shown.



AMPLITUDE = 5

PERIOD = 4 $\frac{1}{4}$ PERIOD = 1

MIDLINE $y=3$ MAX $y=8$

MIN $y=-2$

PHASE SHIFT -3

IMPORTANT POINTS $-2, -1, 0, 1$

ORIENTATION

Prove the identity $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$.

SCORE: ___ / 15 POINTS

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{1 + \sin x}{\cos x} \cdot \frac{1 - \sin x}{1 - \sin x}$$

$$= \frac{1 - \sin^2 x}{\cos x (1 - \sin x)}$$

$$= \frac{\cos^2 x}{\cancel{\cos x}(1 - \sin x)}$$

$$= \frac{\cos x}{1 - \sin x}$$

OR

$$= \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x}$$

$$= \frac{\cos x (1 + \sin x)}{1 - \sin^2 x}$$

$$= \frac{\cancel{\cos x}(1 + \sin x)}{\cos^2 x}$$

$$= \frac{1 + \sin x}{\cos x}$$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x$$