Math 49B (8:30am – 9:20am)
Midterm 1 Version B
Fri Jan 28, 2011

What month is your birthday ? ___ ___
What are the first 2 digits of your address ? ___ ___
What are the last 2 digits of your zip code ? ___ ___
What are the last 2 digits of your DeAnza ID number ? ___ ___

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT

Simplify the expression $\cot\left(\frac{\pi}{2}-x\right)\csc(-x)$. **SCORE: ___ / 8 POINTS**

$= \tan x\,(-\csc x)$

$= \frac{\sin x}{\cos x}\cdot\frac{-1}{\sin x}$

$= -\frac{1}{\cos x} = -\sec x$

Prove the identity $\sec x + \tan x = \frac{\cos x}{1-\sin x}$. **SCORE: ___ / 15 POINTS**

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$= \frac{1+\sin x}{\cos x}\cdot\frac{1-\sin x}{1-\sin x}$

$= \frac{1-\sin^2 x}{\cos x(1-\sin x)}$

$= \frac{\cos^2 x}{\cos x(1-\sin x)}$

$= \frac{\cos x}{1-\sin x}$

OR

$= \frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}$

$= \frac{\cos x(1+\sin x)}{1-\sin^2 x}$

$= \frac{\cos x(1+\sin x)}{\cos^2 x}$

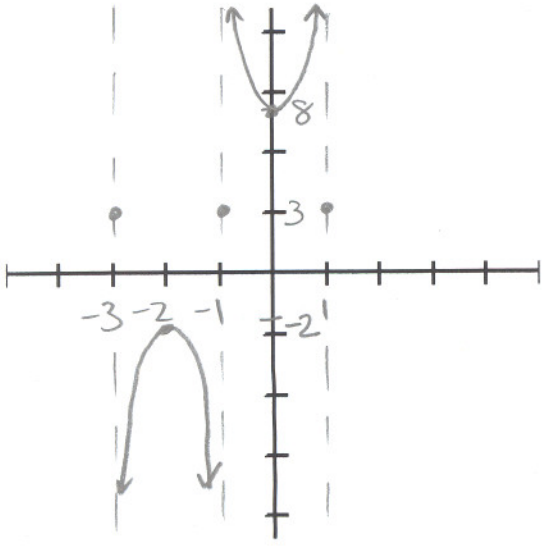
$= \frac{1+\sin x}{\cos x}$

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x$

Graph one period of the function $y = -5\csc\left(\frac{\pi x}{2}+\frac{3\pi}{2}\right)+3$. **SCORE: ___ / 18 POINTS**

Label the important $x-$ and/or $y-$ co-ordinates shown in class.

Draw your graph on the axes included below. Label your axes so the entire graph is shown.



AMPLITUDE = 5
PERIOD = 4 $\frac{1}{4}$ PERIOD = 1
MIDLINE $y=3$ MAX $y=8$
MIN $y=-2$
PHASE SHIFT -3
IMPORTANT POINTS $-2, -1, 0, 1$
ORIENTATION

Use the power-reducing formulae to rewrite $\sin^4 x$ in terms of first powers of cosines. **SCORE: ___ / 12 POINTS**

$$\sin^4 x = \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \frac{1-2\cos 2x+\cos^2 2x}{4}$$

$$= \frac{1-2\cos 2x+\frac{1+\cos 4x}{2}}{4}$$

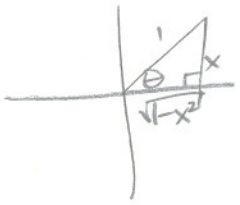
$$= \frac{2-4\cos 2x+1+\cos 4x}{8}$$

$$= \frac{3-4\cos 2x+\cos 4x}{8}$$

If $x > 0$, write $\sin(2\arcsin x)$ as an algebraic expression (ie. an expression without trigonometric functions). **SCORE: ___ / 15 POINTS**

LET $\theta = \arcsin x$

$\sin\theta = x$ AND θ IN Q_1



$$\sin(2\arcsin x) = \sin 2\theta$$
$$= 2\sin\theta\cos\theta$$
$$= 2 \times \sqrt{1-x^2}$$

Simplify $\cos 78^\circ \cos 42^\circ - \sin 78^\circ \sin 42^\circ$. **SCORE: ___ / 6 POINTS**

$$= \cos(78^\circ + 42^\circ)$$
$$= \cos 120^\circ$$
$$= -\frac{1}{2}$$

Find all solutions of the equation $1 + 2\cos\frac{x}{2} = 0$ in the interval $[0, 2\pi)$. **SCORE: ___ / 10 POINTS**

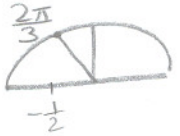
$$\cos\frac{x}{2} = -\frac{1}{2}$$

$$\frac{x}{2} = \frac{2\pi}{3}$$

$$x = \frac{4\pi}{3}$$

$0 \le x < 2\pi$

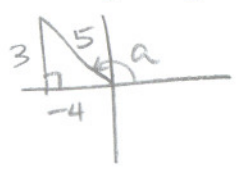
$0 \le \frac{x}{2} < \pi$ ($\frac{1}{2}$ WAY AROUND UNIT CIRCLE)

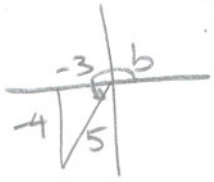


☆ SEE ALSO VERSION A (NOT PREFERRED)

If $\sin a = \frac{3}{5}$ and $\frac{\pi}{2} \le a \le \pi$, and $\sin b = -\frac{4}{5}$ and $\pi \le b \le \frac{3\pi}{2}$, find $\tan(a+b)$.

SCORE: ___ / 15 POINTS





$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$= \frac{-\frac{3}{4} + \frac{4}{3}}{1 - (-\frac{3}{4})(\frac{4}{3})}$$

$$= \frac{\frac{7}{12}}{2} = \frac{7}{24}$$

Find all solutions of the equation $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = -1$.

SCORE: ___ / 18 POINTS

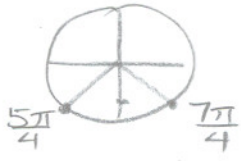
$$\sin x \cos\tfrac{\pi}{4} + \cos x \sin\tfrac{\pi}{4} + \sin x \cos\tfrac{\pi}{4} - \cos x \sin\tfrac{\pi}{4} = -1$$

$$2\sin x \cos\tfrac{\pi}{4} = -1$$

$$(2\sin x)\tfrac{\sqrt{2}}{2} = -1$$

$$\sqrt{2}\sin x = -1$$

$$\sin x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$



$$x = \frac{5\pi}{4} + 2n\pi \text{ OR } \frac{7\pi}{4} + 2n\pi$$

Fill in the blanks. Write DNE if the expression has no value.

SCORE: ___ / 24 POINTS

[a] $\sin^{-1}(-1) =$ $-\frac{\pi}{2}$

[b] $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$ $\frac{3\pi}{4}$

[c] $\tan^{-1}(-\sqrt{3}) =$ $-\frac{\pi}{3}$

[d] $\sin(\arcsin \pi) =$ DNE

[e] $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) =$ $\frac{2\pi}{3}$

[f] $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) =$ $-\frac{\pi}{6}$

[g] The period of $y = \sec x$ is 2π.

[h] The range of $y = \csc x$ is $(-\infty, -1] \cup [1, \infty)$.

[i] The period of $y = \cot x$ is π.

[j] The domain of $y = \sec x$ is $\{x \neq \frac{\pi}{2} + n\pi \mid n \in \mathbb{Z}\}$.

[k] The domain of $y = \tan^{-1} x$ is $(-\infty, \infty)$.

[l] The range of $y = \cos^{-1} x$ is $[0, \pi]$.

[m] The **equations** of the asymptotes of $y = \cot x$ are $x = n\pi, n \in \mathbb{Z}$.

Graph $y = \tan^{-1} x$. Label the important $x-$ and/or $y-$ co-ordinates shown in class. **SCORE: ___ / 9 POINTS**

Draw your graph on the axes included below.