Math 49B (8:30am – 9:20am)
Midterm 2 Version A
Fri Feb 25, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 150 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT

Point P lies below the xy – plane, in front of the yz – plane, and to the left of the xz – plane. In which octant does P lie ? **SCORE: ___ / 3 POINTS**

$z<0,\ x>0,\ y<0$ QUADRANT 4 + 4 = OCTANT 8

A conic has eccentricity $\frac{2}{3}$ and directrix $y=-4$. **SCORE: ___ / 9 POINTS**

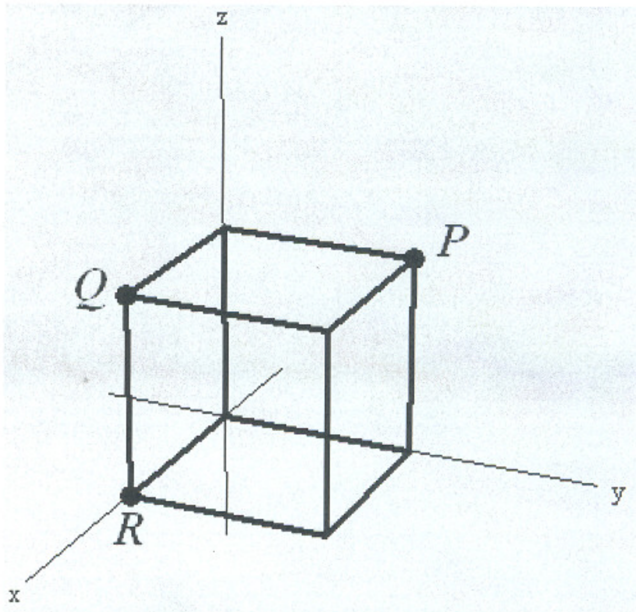
[a] What type of conic is it ?

$e<1 \rightarrow$ ELLIPSE

[b] Find the polar equation of the conic.
SIMPLIFY YOUR FINAL ANSWER SO THAT ALL COEFFICIENTS ARE INTEGERS.

$$r=\frac{ep}{1-e\sin\theta}=\frac{\frac{2}{3}(4)}{1-\frac{2}{3}\sin\theta}\cdot\frac{3}{3}=\frac{8}{3-2\sin\theta}$$

The diagram on the right shows a cube with one corner at the origin, and 3 edges along the co-ordinate axes. Each side of the cube is 3 units long. **SCORE: ___ / 20 POINTS**



[a] Find the co-ordinates of P, Q and R.

$P =$ (0, 3, 3)

$Q =$ (3, 0, 3)

$R =$ (3, 0, 0)

[b] **Using vectors**, find $\angle PRQ$.
Your final answer may involve trigonometric or inverse trigonometric functions.

$\overrightarrow{RP}=\langle -3,3,3\rangle$

$\overrightarrow{RQ}=\langle 0,0,3\rangle$

$$\cos\theta=\frac{\overrightarrow{RP}\cdot\overrightarrow{RQ}}{\|\overrightarrow{RP}\|\,\|\overrightarrow{RQ}\|}=\frac{9}{(3\sqrt{3})(3)}=\frac{1}{\sqrt{3}}$$

$$\theta=\cos^{-1}\frac{1}{\sqrt{3}}$$

Your quiz partner ran all the symmetry tests on a polar equation, but only told you some of the results. **SCORE: ___ / 9 POINTS**

When he replaced (r, θ) with $(-r, -\theta)$, he did not get the original equation.

When he replaced (r, θ) with $(r, \pi - \theta)$ or $(-r, \theta)$, he got the original equation.

[a] Which types of symmetry was your quiz partner testing for that gave the results above ?

$(-r, -\theta) \rightarrow \theta = \frac{\pi}{2}$

$(r, \pi - \theta) \rightarrow \theta = \frac{\pi}{2}$

$(-r, \theta) \rightarrow$ POLE

[b] Based on the results of his tests, what can you conclude about the symmetry of the graph ?

THE GRAPH IS SYMMETRIC OVER $\theta = \frac{\pi}{2}$,
THE POLE
AND THE POLAR AXIS

By eliminating the parameter, you can show that the parametric equations $\begin{matrix} x = \cos t \\ y = 2\cos t + 1 \end{matrix}$ and $\begin{matrix} x = e^t \\ y = 2e^t + 1 \end{matrix}$ **SCORE: ___ / 9 POINTS**

correspond to the same rectangular equation.

[a] Find that rectangular equation.

$y = 2x + 1$

[b] Describe how the plane curves represented by the two sets of parametric equations are different from each other.

THE FIRST CURVE ONLY INCLUDES POINTS BETWEEN
(-1, -1) AND (1, 3)
THE SECOND CURVE CONTAINS ALL POINTS WHERE $x > 0$
AND $y > 1$

Convert the polar equation $r^2 = \sin 2\theta$ to rectangular. **<u>HINT: Use the double angle formula for</u>** $\sin 2\theta$**.** **SCORE: ___ / 9 POINTS**

$r^2 = 2\sin\theta\cos\theta$

$r^2 = 2\left(\frac{y}{r}\right)\left(\frac{x}{r}\right)$

$r^4 = 2xy$

$(x^2+y^2)^2 = 2xy$

Find parametric equations of the ellipse with vertices $(\pm 4, 0)$ and foci $(\pm 3, 0)$. **SCORE: ___ / 9 POINTS**

$3^2 + b^2 = 4^2$

$b = \sqrt{7}$

$x = 4\cos t$

$y = \sqrt{7}\sin t$

The line segment connecting $(1, -5, 2)$ and $(-11, -1, 8)$ is a diameter of a sphere. Find the equation of the sphere.

SCORE: ___ / 15 POINTS

CENTER $= \left(\frac{1+-11}{2}, \frac{-5+-1}{2}, \frac{2+8}{2}\right) = (-5, -3, 5)$

RADIUS $= \sqrt{(1--5)^2+(-5--3)^2+(2-5)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

$(x+5)^2+(y+3)^2+(z-5)^2 = 49$

Write $\vec{w} = \langle 2, 2 \rangle$ as the sum of two orthogonal vectors, one of which is the projection of $\vec{w}$ onto $\vec{r} = \langle 6, 1 \rangle$.

SCORE: ___ / 15 POINTS

$\text{PROJ}_{\vec{r}}\vec{w} = \frac{\vec{w}\cdot\vec{r}}{\vec{r}\cdot\vec{r}}\vec{r} = \frac{14}{37}\langle 6, 1\rangle = \left\langle \frac{84}{37}, \frac{14}{37} \right\rangle$

$\vec{w} - \text{PROJ}_{\vec{r}}\vec{w} = \langle 2, 2 \rangle - \left\langle \frac{84}{37}, \frac{14}{37} \right\rangle = \left\langle \frac{-10}{37}, \frac{60}{37} \right\rangle$

$\langle 2, 2 \rangle = \left\langle \frac{84}{37}, \frac{14}{37} \right\rangle + \left\langle \frac{-10}{37}, \frac{60}{37} \right\rangle$

Find a **unit** vector orthogonal to both $\langle 2, -1, -3 \rangle$ and $\langle -1, 3, -2 \rangle$.

SCORE: ___ / 15 POINTS

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -3 \\ -1 & 3 & -2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 2 & -1 \\ -1 & 3 \end{matrix} = \begin{matrix} 2\vec{i} + 3\vec{j} + 6\vec{k} \\ +9\vec{i} + 4\vec{j} - \vec{k} \end{matrix} = \langle 11, 7, 5 \rangle$$

$\frac{1}{\|\langle 11, 7, 5\rangle\|}\langle 11, 7, 5 \rangle = \frac{1}{\sqrt{195}}\langle 11, 7, 5 \rangle$

Find $(-\sqrt{3}+i)^9$ using DeMoivre's Theorem.

SCORE: ___ / 20 POINTS

$r = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$

$\theta = \pi + \tan^{-1}\frac{1}{-\sqrt{3}} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\left(2 \operatorname{cis} \frac{5\pi}{6}\right)^9 = 2^9 \operatorname{cis} 9\left(\frac{5\pi}{6}\right)$

$= 512\left(\cos\frac{15\pi}{2} + i\sin\frac{15\pi}{2}\right)$

$= 512(0 + i(-1))$

$= -512i$

Find the endpoints of both latera recta of the conic with polar equation $r = \dfrac{16}{3+5\sin\theta}$. **SCORE: ___ / 20 POINTS**

θ	r	RECTANGULAR	
0	$\frac{16}{3}$	$(\frac{16}{3}, 0)$	LR
$\frac{\pi}{2}$	$\frac{16}{8} = 2$	$(0, 2)$	V
π	$\frac{16}{3}$	$(-\frac{16}{3}, 0)$	LR
$\frac{3\pi}{2}$	$\frac{16}{-2} = -8$	$(0, 8)$	V

$$\text{CENTER} = \left(\frac{0+0}{2}, \frac{2+8}{2}\right) = (0, 5)$$

$$\text{FOCUS} = (0, 10)$$

$$\left(\pm\frac{16}{3}, 0\right), \left(\pm\frac{16}{3}, 10\right)$$