

Graphing the conic with polar equation $r = \frac{a}{b + c \cos \theta}$ or $r = \frac{a}{b - c \cos \theta}$ or $r = \frac{a}{b + c \sin \theta}$ or $r = \frac{a}{b - c \sin \theta}$

All polar equations of the above four types correspond to conics with the pole as a/the focus

- [1] Multiply numerator and denominator of the polar equation by $\frac{1}{b}$ to get a constant term of 1 in the denominator

The equation then becomes $r = \frac{A}{1 + e \cos \theta}$ or $r = \frac{A}{1 - e \cos \theta}$ or $r = \frac{A}{1 + e \sin \theta}$ or $r = \frac{A}{1 - e \sin \theta}$

- [2] The eccentricity (e) is the absolute value of the coefficient of the trigonometric function in the denominator.
 If $e = 1$, the conic is a parabola.
 If $0 < e < 1$, the conic is an ellipse.
 If $e > 1$, the conic is a hyperbola.

The numerator (A) is the eccentricity (e) multiplied by the distance from the pole/focus to the directrix (p).

$$A = ep, \text{ so } p = \frac{A}{e}.$$

If the equation involves $\cos \theta$ in the denominator, then the directrix is vertical ($x = p$).

If the equation involves $\sin \theta$ in the denominator, then the directrix is horizontal ($y = p$).

If the coefficient of the trigonometric function in the denominator is positive,
 the directrix is to the right of ($x = p$) or above ($y = p$) the pole/focus.

If the coefficient of the trigonometric function in the denominator is negative,
 the directrix is to the left of ($x = -p$) or below ($y = -p$) the pole/focus.

The directrix is NEVER an axis of symmetry.

Part of the conic lies between the pole/focus and the directrix.

That part of the conic always curves around the pole/focus away from the directrix.

- [3] Plot the points corresponding to $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

These are the x – and y – intercepts of the conic.

NOTE: If the conic is a parabola, one of the four points will NOT exist.

The latus rectum passes through the pole/focus, and connects the two points above which are reflections of each other through the pole/focus.

That is, the two points whose rectangular co-ordinates are negatives of each other.

The other point (points) is (are) the vertex (vertices).

If the conic is an ellipse or a hyperbola:

- [4] The center is the midpoint of the vertices. **The pole/focus is NEVER the center.**

- [5] The center is also the midpoint of the two foci.

Double the co-ordinates of the center to get the other focus.

- [6] The other latus rectum passes through the other focus and is symmetric to the first latus rectum.

The ends of the other latus rectum share a non-zero co-ordinate with the other focus,
 and a non-zero co-ordinate with the ends of the first latus rectum.

- [7] Use the vertices and the ends of the latera recta to sketch the conic.

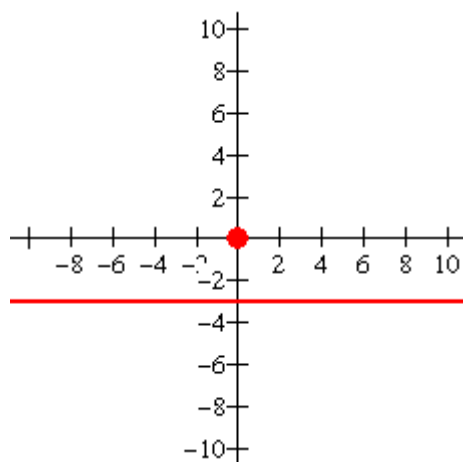
Example: Graphing the conic with polar equation $r = \frac{12}{2 - 4\sin \theta}$

$$[1] \quad r = \frac{12}{2 - 4\sin \theta} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{6}{1 - 2\sin \theta}$$

$$[2] \quad e = |-2| = 2 > 1 \rightarrow \text{hyperbola}$$

$$6 = ep = 2p, \text{ so } p = 3.$$

Since the equation involves $\sin \theta$ in the denominator,
and the coefficient of $\sin \theta$ in the denominator is negative,
therefore the directrix is horizontal and below the pole/focus at $y = -3$.

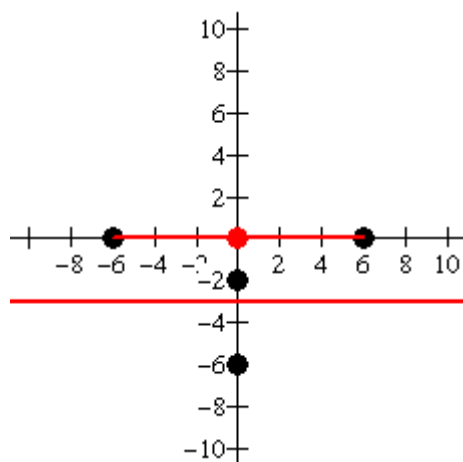


[3]

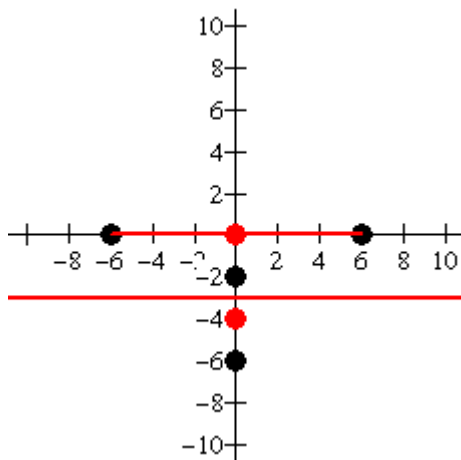
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$r = \frac{12}{2 - 4\sin \theta}$	6	-6	6	2
(x, y)	(6, 0)	(0, -6)	(-6, 0)	(0, -2)

The latus rectum connects (6, 0) and (-6, 0).

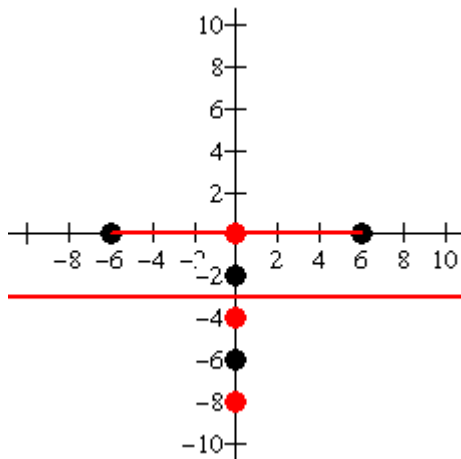
The vertices are (0, -6) and (0, -2).



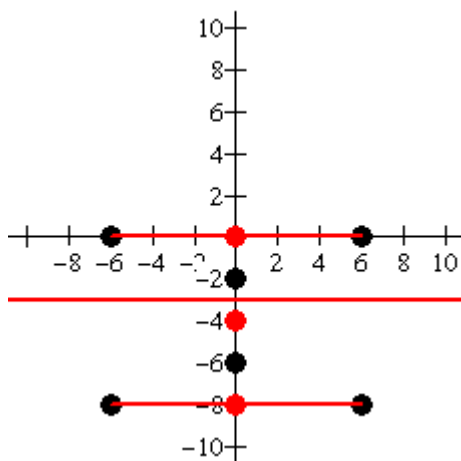
[4] The center is $\left(\frac{0+0}{2}, \frac{-6+-2}{2}\right) = (0, -4)$.



[5] The other focus is $(2 \times 0, 2 \times -4) = (0, -8)$.



[6] The other latus rectum passes through $(0, -8)$, $(6, -8)$ and $(-6, -8)$.



[7] Final result

