AN EFFICIENT WAY TO GRAPH MANY POLAR FUNCTIONS

To graph $r = f(\theta)$:

1. Run symmetry tests to determine the minimum interval of the graph you need to plot first.

2. Solve $f(\theta) = 0$ to determine the angles in the minimum interval at which the graph goes through the pole. At those points, the graph will also be tangent to the line through the pole at that angle.

3. Find the maximum and minimum values of $f(\theta)$ to determine when the graph changes from spiraling outward (away from the pole) to spiraling inward (towards the pole), or vice versa.

4. Find the quarter period points of $f(\theta)$ (phase shift $\pm\ n \times$(¼ period)) that fall within the minimum interval.

5. Plot the points and tangent lines in steps 2 and 4, and use spiraling to connect them.

6. Reflect the graph in step 5 into the other quadrants according to the results of the symmetry tests.

SYMMETRY TESTS

Symmetry of graph Polar terminology	**Symmetry of graph Rectangular terminology**	**Tests**	**Minimum Interval**
over the polar axis	over the x-axis	$(r, -\theta)$ $(-r, \pi-\theta)$	$\theta \in [0, \pi]$
over $\theta = \frac{\pi}{2}$	over the y-axis	$(r, \pi-\theta)$ $(-r, -\theta)$	$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
about the pole	through the origin	$(r, \pi+\theta)$ $(-r, \theta)$	either $\theta \in [0, \pi]$ or $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Notes regarding symmetry tests:

A. Run each symmetry test by replacing (r, θ) in the function with the corresponding ordered pair above.
If the equation can then be simplified to the original function, then the graph has that type of symmetry.
If the equation cannot be simplified to the original function, then the test has no conclusion.
The result of a symmetry test is NEVER "the graph is not symmetric".

B. Each symmetry test actually has infinitely many variations, only two of which are listed above.
The other variations result from replacing the angle in the test with every co-terminal angle.
For this class, we will only run the symmetry tests above.

C. If a function passes one version of a symmetry test, it is not necessary to run the other version.

D. If a function has two of the three types of symmetry, it automatically has all three types.
In that case, it not necessary to run the third set of symmetry tests, and the minimum interval becomes $\theta \in \left[0, \frac{\pi}{2}\right]$.

Example: Graph $r = \sin 2\theta$

1. Symmetry over polar axis

$r = \sin 2(-\theta)$	$-r = \sin 2(\pi - \theta)$
$r = -\sin 2\theta$	$-r = \sin(2\pi - 2\theta)$
NO CONCLUSION	$-r = \sin 2\pi \cos 2\theta - \cos 2\pi \sin 2\theta$
	$-r = -\sin 2\theta$
	$r = \sin 2\theta$
	SYMMETRIC OVER POLAR AXIS

Symmetry about pole

$r = \sin 2(\pi + \theta)$

$r = \sin(2\pi + 2\theta)$

$r = \sin 2\pi \cos 2\theta + \cos 2\pi \sin 2\theta$

$r = \sin 2\theta$

SYMMETRIC ABOUT POLE – skip other test

Since the graph has the above two symmetries, it also has the third symmetry ie. it is also symmetric over $\theta = \frac{\pi}{2}$.

So, the minimum interval we need to plot first is $\theta \in \left[0, \frac{\pi}{2}\right]$.

2. $\sin 2\theta = 0$ for $\theta \in \left[0, \frac{\pi}{2}\right]$ when $\theta = 0, \frac{\pi}{2}$

The graph will pass through the pole when $\theta = 0, \frac{\pi}{2}$ and will be tangent to the lines corresponding to $\theta = 0, \frac{\pi}{2}$

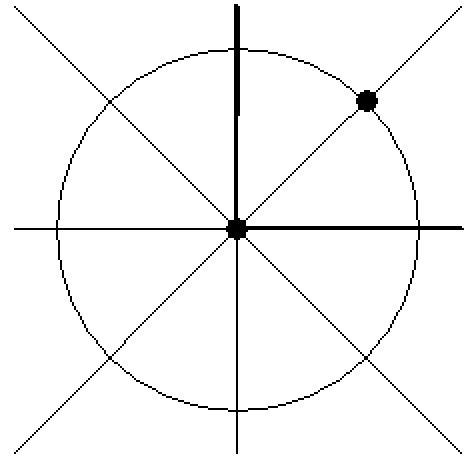
3. The amplitude of $\sin 2\theta$ is 1, and the midline is 0, so the maximum is 1 and the minimum is -1.

4. The period of $\sin 2\theta$ is π, and the phase shift is 0, so the ¼ period is $\frac{\pi}{4}$.

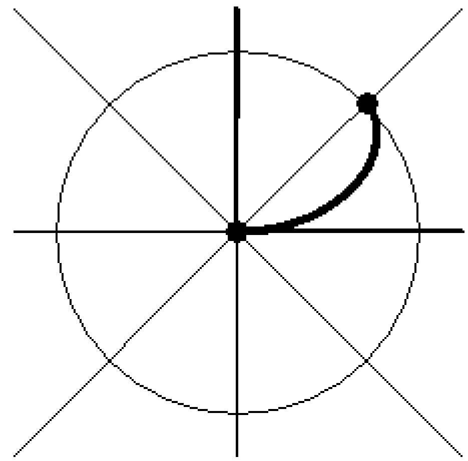
The ¼ period points in $\left[0, \frac{\pi}{2}\right]$ are $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$.

θ	$r = \sin 2\theta$
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0

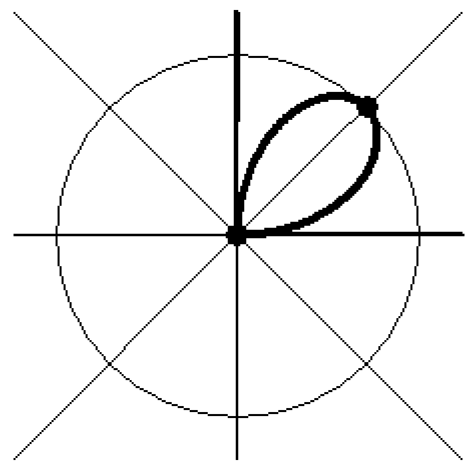
5. Plotting the two tangent lines from step 2
and the three points from step 4 (two of which are at the pole) gives



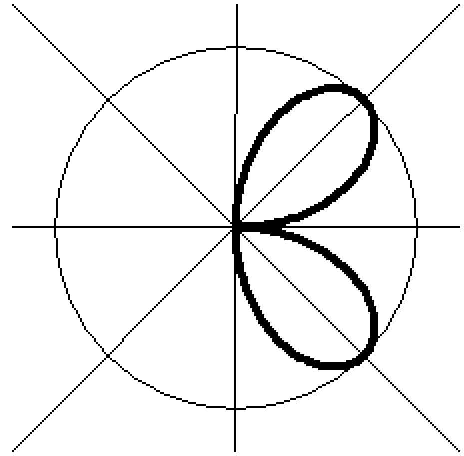
Starting at $\theta = 0$ [the polar point $(0, 0)$],
connect to $\theta = \frac{\pi}{4}$ [the polar point $\left(1, \frac{\pi}{4}\right)$] by spiraling counterclockwise.
Since the first point $(0, 0)$ is closer to the pole
than the second point $\left(1, \frac{\pi}{4}\right)$, we need to spiral outward.



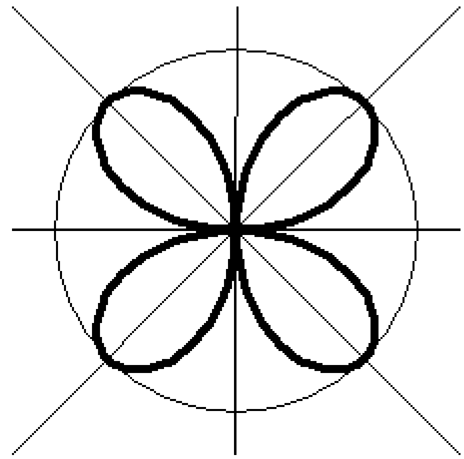
Continuing from $\theta = \frac{\pi}{4}$ [the polar point $\left(1, \frac{\pi}{4}\right)$],
connect to $\theta = \frac{\pi}{2}$ [the polar point $\left(0, \frac{\pi}{2}\right)$] by spiraling counterclockwise.
Since the first point $\left(1, \frac{\pi}{4}\right)$ is farther from the pole
than the second point $\left(0, \frac{\pi}{2}\right)$, we need to spiral inward.



6. Since the graph is symmetric over the polar axis, reflect the partial graph vertically.



Since the graph is also symmetric over $\theta = \frac{\pi}{2}$, reflect the new partial graph horizontally.



The graph is now complete.