

SCORE: ____ / 20 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW PROPER WORK TO RECEIVE FULL CREDIT

Use the trigonometric substitution $x = 3 \csc \theta$

SCORE: ____ / 3 POINTS

to write the algebraic function $\sqrt{4x^2 - 36}$ as a trigonometric function. Simplify your answer. (Assume $0 < \theta \leq \frac{\pi}{2}$.)

$$\begin{aligned} & \sqrt{4(3 \csc \theta)^2 - 36} \\ &= \sqrt{36 \csc^2 \theta - 36} \\ &= 6 \sqrt{\csc^2 \theta - 1} \\ &= 6 \sqrt{\cot^2 \theta} \\ &= 6 \cot \theta \end{aligned}$$

Prove that $\frac{\cot x \cos x}{1 - \sin x} - 1 = \csc x$.

SCORE: ____ / 6 POINTS

$$\begin{aligned} & \frac{\cot x \cos x}{1 - \sin x} - 1 \\ &= \frac{\frac{\cos x}{\sin x} \cdot \cos x}{1 - \sin x} - 1 \\ &= \frac{\cos^2 x}{\sin x (1 - \sin x)} - 1 \\ &= \frac{\cos^2 x - \sin x (1 - \sin x)}{\sin x (1 - \sin x)} \\ &= \frac{\cos^2 x - \sin x + \sin^2 x}{\sin x (1 - \sin x)} \\ &= \frac{\cos^2 x + \sin^2 x - \sin x}{\sin x (1 - \sin x)} \\ &= \frac{1 - \sin x}{\sin x (1 - \sin x)} \\ &= \frac{1}{\sin x} = \csc x \end{aligned}$$

$-\frac{1}{2}$ IF FORGOTTEN



If $\sec x = \frac{3}{2}$ and $\tan x < 0$, find $\cot x$ using the fundamental identities (NOT TRIANGLES).

SCORE: ___ / 3 POINTS

$$\begin{aligned}\tan^2 x &= \sec^2 x - 1 \\ &= \left(\frac{3}{2}\right)^2 - 1 \\ &= \frac{5}{4}\end{aligned}$$

OR

$$\tan x = -\frac{\sqrt{5}}{2}$$

$$\cot x = \frac{1}{\tan x} = -\frac{2}{\sqrt{5}}$$

Using the fundamental identities, simplify $\cos(-m)\sec\left(\frac{\pi}{2} - m\right)$.

SCORE: ___ / 2 POINTS

$$\begin{aligned}&= \cos m \csc m \\ &= \cos m \frac{1}{\sin m} \\ &= \frac{\cos m}{\sin m} \quad \text{OR} \\ &= \cot m\end{aligned}$$

Prove that $\cot x - \sec x \csc x = -\tan x$.

SCORE: ___ / 6 POINTS

$$= \frac{\cos x}{\sin x} - \frac{1}{\cos x} \frac{1}{\sin x}$$

$$= \frac{\cos^2 x - 1}{\sin x \cos x}$$

$$= \frac{-\sin^2 x}{\sin x \cos x}$$

$$= \frac{-\sin x}{\cos x}$$

OR JUST CANCEL $\sin x$
IN PREVIOUS STEP

$$= -\tan x$$

← -1/2 IF FORGOTTEN