

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT

Give polar co-ordinates for the point with rectangular co-ordinates $(-1, \sqrt{3})$.

$$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

$$\theta = \pi + \arctan(\sqrt{3}) = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

SUBTRACT $\frac{1}{2}$ POINT
 IF YOU DIDN'T USE
 (,) NOTATION

$$(r, \theta) = (2, \frac{2\pi}{3})$$

SCORE: ____ / 3 POINTS

Use DeMoivre's Theorem to find $(-1 + i\sqrt{3})^5$.

SCORE: ____ / 5 POINTS

Use trigonometric form for all work, but write your final answer in standard form.

USING WORK FROM QUESTION ABOVE $-1 + i\sqrt{3} = 2 \text{cis } \frac{2\pi}{3}$

$$\begin{aligned} (2 \text{cis } \frac{2\pi}{3})^5 &= 2^5 \text{cis } 5(\frac{2\pi}{3}) = 32 \text{cis } \frac{10\pi}{3} \\ &= 32 [\cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3}] \\ &= 32 [-\frac{1}{2} - \frac{\sqrt{3}}{2}i] = -16 - 16\sqrt{3}i \end{aligned}$$

A point has polar co-ordinates $(8, \frac{5\pi}{3})$.

SCORE: ____ / 4 POINTS

[a] Find a polar representation of the point using a positive r -value and a negative θ -value.

$$(8, \frac{5\pi}{3} - 2\pi) = (8, -\frac{\pi}{3})$$

[b] Find a polar representation of the point using a negative r -value and a positive θ -value.

$$(-8, \frac{5\pi}{3} - \pi) = (-8, \frac{2\pi}{3})$$

[c] Find the rectangular co-ordinates of the point.

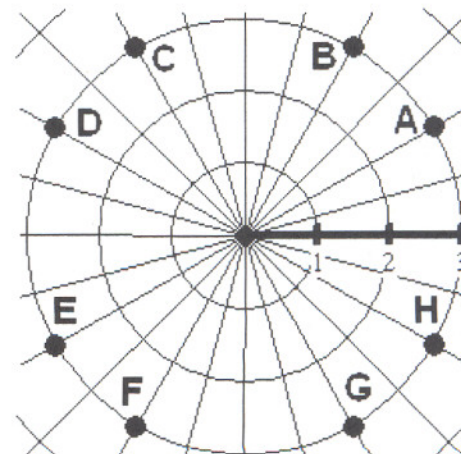
$$(8 \cos \frac{5\pi}{3}, 8 \sin \frac{5\pi}{3}) = (8(\frac{1}{2}), 8(-\frac{\sqrt{3}}{2})) = (4, -4\sqrt{3})$$

Fill in the blanks using the graph on the right.

SCORE: ____ / 2 POINTS

[a] The polar co-ordinates $(-3, \frac{5\pi}{6})$ refers to point H.

[b] The polar co-ordinates $(-3, -\frac{2\pi}{3})$ refers to point B.



Find $\frac{18(\cos 84^\circ + i \sin 84^\circ)}{3(\cos 31^\circ + i \sin 31^\circ)}$. Write your final answer in trigonometric form.

SCORE: ___ / 2 POINTS

$$\frac{18}{3} \text{cis}(84^\circ - 31^\circ) = \underline{6 \text{cis} 53^\circ}$$

If $x > 0$, write $\cos(2\cos^{-1} x)$ as an algebraic expression (ie. an expression without trigonometric functions). SCORE: ___ / 3 POINTS

$$\text{LET } \theta = \cos^{-1} x$$

$$\cos \theta = x$$

$$\cos(2\cos^{-1} x) = \cos 2\theta = 2\cos^2 \theta - 1 = \underline{2x^2 - 1}$$

Convert the polar equation $r = 1 - 2\cos \theta$ to rectangular form.

SCORE: ___ / 5 POINTS

Simplify your answer so that it has no fractions, radicals or negative/fractional exponents.

$$\begin{aligned} r &= 1 - 2\left(\frac{x}{r}\right) \\ r^2 &= r - 2x \\ x^2 + y^2 &= \sqrt{x^2 + y^2} - 2x \\ x^2 + y^2 + 2x &= \sqrt{x^2 + y^2} \\ (x^2 + y^2 + 2x)^2 &= x^2 + y^2 \end{aligned}$$

$$\begin{aligned} r^2 &= r - 2r\cos \theta \\ x^2 + y^2 &= \sqrt{x^2 + y^2} - 2x \\ x^2 + y^2 + 2x &= \sqrt{x^2 + y^2} \\ (x^2 + y^2 + 2x)^2 &= x^2 + y^2 \end{aligned}$$

Test $r = 1 - \cos \theta$ for symmetry with respect to the pole.

SCORE: ___ / 6 POINTS

State clearly what the results of your test tell you about the symmetry of this particular graph.

$$\frac{1}{2} r = 1 - \cos(\pi + \theta)$$

$$r = 1 - [\cos \pi \cos \theta - \sin \pi \sin \theta]$$

$$r = 1 + \cos \theta$$

NO CONCLUSION / DON'T KNOW

$$\frac{1}{2} -r = 1 - \cos \theta$$

$$r = -1 + \cos \theta$$

NO CONCLUSION / DON'T KNOW

THE TEST DOESN'T TELL US IF THE GRAPH IS SYMMETRIC OR NOT - THERE IS NO CONCLUSION