

SCORE: ____ / 30 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT

Give polar co-ordinates for the point with rectangular co-ordinates $(-\sqrt{3}, 1)$.

SUBTRACT $\frac{1}{2}$ POINT
 IF YOU DIDN'T USE
 $(,)$ NOTATION

SCORE: ____ / 3 POINTS

$$r = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$$

$$\theta = \pi + \arctan\left(\frac{1}{-\sqrt{3}}\right) = \pi + \frac{-\pi}{6} = \frac{5\pi}{6}$$

$$(r, \theta) = \left(2, \frac{5\pi}{6}\right)$$

Use DeMoivre's Theorem to find $(-\sqrt{3} + i)^5$.

SCORE: ____ / 5 POINTS

Use trigonometric form for all work, but write your final answer in standard form.

USING WORK FROM QUESTION ABOVE $-\sqrt{3} + i = 2 \operatorname{cis} \frac{5\pi}{6}$

$$(2 \operatorname{cis} \frac{5\pi}{6})^5 = 2^5 \operatorname{cis} 5(\frac{5\pi}{6}) = 32 \operatorname{cis} \frac{25\pi}{6}$$

$$= 32 \left[\cos \frac{25\pi}{6} + i \sin \frac{25\pi}{6} \right]$$

$$= 32 \left[\frac{\sqrt{3}}{2} + \frac{1}{2}i \right] = 16\sqrt{3} + 16i$$

A point has polar co-ordinates $\left(8, \frac{7\pi}{6}\right)$.

SCORE: ____ / 4 POINTS

[a] Find a polar representation of the point using a positive r -value and a negative θ -value.

$$\left(8, \frac{7\pi}{6} - 2\pi\right) = \left(8, -\frac{5\pi}{6}\right)$$

[b] Find a polar representation of the point using a negative r -value and a positive θ -value.

$$\left(-8, \frac{7\pi}{6} - \pi\right) = \left(-8, \frac{\pi}{6}\right)$$

[c] Find the rectangular co-ordinates of the point.

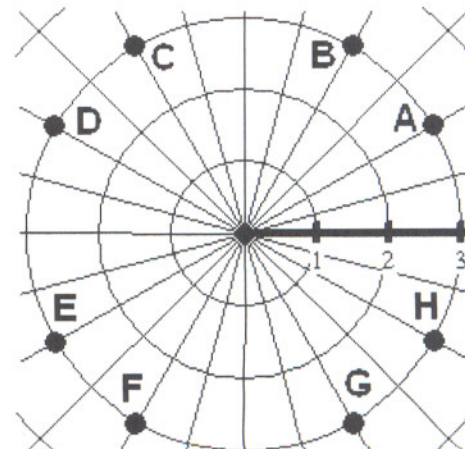
$$\left(8 \cos \frac{7\pi}{6}, 8 \sin \frac{7\pi}{6}\right) = \left(8\left(-\frac{\sqrt{3}}{2}\right), 8\left(-\frac{1}{2}\right)\right) = (-4\sqrt{3}, -4)$$

Fill in the blanks using the graph on the right.

SCORE: ____ / 2 POINTS

[a] The polar co-ordinates $\left(-3, -\frac{5\pi}{6}\right)$ refers to point A.

[b] The polar co-ordinates $\left(-3, \frac{2\pi}{3}\right)$ refers to point G.



Find $\frac{24(\cos 97^\circ + i \sin 97^\circ)}{8(\cos 62^\circ + i \sin 62^\circ)}$. Write your final answer in trigonometric form.

SCORE: ___ / 2 POINTS

$$\frac{24}{8} \operatorname{cis}(97^\circ - 62^\circ) = \underline{3 \operatorname{cis} 35^\circ}$$

If $x > 0$, write $\tan(2 \tan^{-1} x)$ as an algebraic expression (ie. an expression without trigonometric functions).

SCORE: ___ / 3 POINTS

$$\text{LET } \theta = \tan^{-1} x$$

$$\tan \theta = x$$

$$\tan(2 \tan^{-1} x) = \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \underline{\frac{2x}{1-x^2}} \quad 3$$

Convert the polar equation $r = 1 - 3 \sin \theta$ to rectangular form.

SCORE: ___ / 5 POINTS

Simplify your answer so that it has no fractions, radicals or negative/fractional exponents.

$$\underline{r = 1 - 3\left(\frac{y}{r}\right)} \quad \frac{1}{2}$$

$$\underline{r^2 = r - 3y} \quad \frac{1}{2}$$

$$\underline{x^2 + y^2 = \sqrt{x^2 + y^2} - 3y} \quad 3$$

$$x^2 + y^2 + 3y = \sqrt{x^2 + y^2}$$

$$\underline{(x^2 + y^2 + 3y)^2 = x^2 + y^2} \quad 1$$

OR

$$\underline{r^2 = r - 3r \sin \theta} \quad 1$$

$$\underline{x^2 + y^2 = \sqrt{x^2 + y^2} - 3y} \quad 3$$

$$x^2 + y^2 + 3y = \sqrt{x^2 + y^2}$$

$$\underline{(x^2 + y^2 + 3y)^2 = x^2 + y^2} \quad 1$$

Test $r = 1 - \cos \theta$ for symmetry with respect to $\theta = \frac{\pi}{2}$.

SCORE: ___ / 6 POINTS

State clearly what the results of your test tell you about the symmetry of this particular graph.

$$\frac{1}{2} \underline{r = 1 - \cos(\pi - \theta)}$$

$$r = 1 - [\cos \pi \cos \theta + \sin \pi \sin \theta]$$

$$\underline{r = 1 + \cos \theta}$$

NO CONCLUSION / DON'T KNOW

$$\frac{1}{2} \underline{-r = 1 - \cos(-\theta)}$$

$$-r = 1 - \cos \theta$$

$$\underline{r = -1 + \cos \theta}$$

NO CONCLUSION / DON'T KNOW

THE TEST DOES NOT TELL US IF THE GRAPH IS SYMMETRIC OR NOT - THERE IS NO CONCLUSION 1