

SCORE: \_\_\_\_ / 30 POINTS

**NO CALCULATORS ALLOWED****YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT**Use the power reducing formulae to rewrite  $\cos^4 x$  in terms of the first powers of cosines.

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$$\begin{aligned}
 \cos^4 x &= (\cos^2 x)^2 \\
 &= \left( \frac{1 + \cos 2x}{2} \right)^2 \\
 &= \frac{1 + 2\cos 2x + \cos^2 2x}{4} \\
 &= \frac{1 + 2\cos 2x + \frac{1 + \cos 4x}{2}}{4} = \frac{3 + 4\cos 2x + \cos 4x}{8}
 \end{aligned}$$

Graph the conic with polar equation  $r = \frac{5}{3 - 2\cos\theta}$  by answering the following questions first.

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You must use techniques discussed either in lecture or in the handouts.

- [a] Determine the eccentricity and type of the conic.

$$r = \frac{\frac{5}{3}}{1 - \frac{2}{3}\cos\theta} \quad e = \frac{2}{3} \Rightarrow \text{ELLIPSE}$$

- [b] Determine the equation of the directrix.

$$ep = \frac{5}{3} \Rightarrow \frac{2}{3}p = \frac{5}{3} \Rightarrow p = \frac{5}{2} \quad x = -\frac{5}{2}$$

- [c] Determine the rectangular co-ordinates of the
- $x$
- and
- $y$
- intercepts.

$\theta$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$(5, 0)$	$(-1, 0)$
$r$	5	$\frac{5}{3}$	1	$\frac{5}{3}$	$(0, \frac{5}{3})$	$(0, -\frac{5}{3})$

- [d] Determine the rectangular co-ordinates of the vertices.

$$(5, 0) \quad (-1, 0)$$

- [e] Determine the rectangular co-ordinates of both foci.

$$\text{CENTER} = \left( \frac{5 + (-1)}{2}, \frac{0 + 0}{2} \right) = (2, 0)$$

$$\text{FOCUS} = (4, 0) \text{ AND } (0, 0)$$

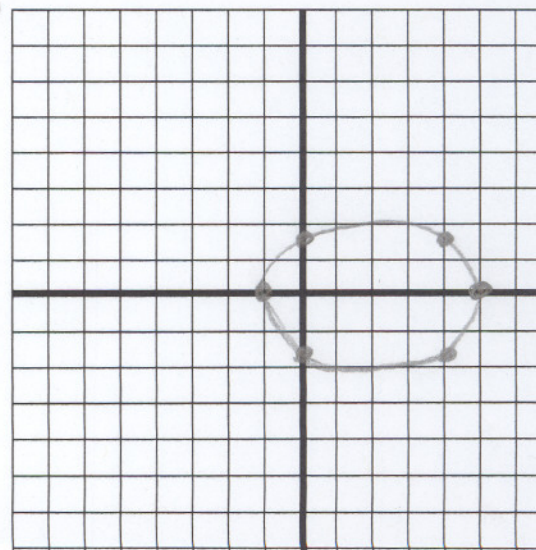
- [f] Determine the rectangular co-ordinates of the endpoints of both latera recta.

$$(0, \frac{5}{3}) \quad (0, -\frac{5}{3}) \quad (4, \frac{5}{3}) \quad (4, -\frac{5}{3})$$

- [g] Plot the points you found which lie on the conic, and then sketch the conic.

 $\frac{1}{2}$  POINT EACH

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Graph the polar function  $r = 1 + 2 \cos 2\theta$  by answering the following questions first.  
You must use techniques discussed either in lecture or in the handouts.

SCORE: \_\_\_ / 16 POINTS

- [a] Determine if the graph is symmetric over the polar axis.

$$r = 1 + 2 \cos 2(-\theta)$$

$$r = 1 + 2 \cos 2\theta$$

SYMMETRIC

- [b] Determine if the graph is symmetric about the pole.

$$r = 1 + 2 \cos 2(\pi + \theta)$$

$$r = 1 + 2 \cos (2\pi + 2\theta)$$

$$r = 1 + 2(\cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta)$$

$$\frac{1}{2} r = 1 + 2 \cos 2\theta \quad \text{SYMMETRIC}$$

- [c] Determine if the graph is symmetric over  $\theta = \frac{\pi}{2}$ .

BECAUSE THE GRAPH IS SYMMETRIC OVER THE POLAR AXIS AND THE POLE, IT IS AUTOMATICALLY SYMMETRIC OVER  $\theta = \frac{\pi}{2}$ .

- [d] What is the minimum interval of the graph that must be plotted first?

$$\left[0, \frac{\pi}{2}\right]$$

- [e] Find all values of  $\theta$  within the minimum interval at which the graph passes through the pole.

$$1 + 2 \cos 2\theta = 0 \quad \text{FOR } 0 \leq \theta \leq \frac{\pi}{2} \Rightarrow 0 \leq 2\theta \leq \pi$$

$$\cos 2\theta = -\frac{1}{2}$$

$$2\theta = \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

- [f] Find the maximum and minimum values of  $r$ .

$$\text{AMPLITUDE} = 2 \quad \text{MAX} = 3$$

$$\text{MIDLINE} = 1 \quad \text{MIN} = -1$$

- [g] Find  $r$  and  $\theta$  for all the quarter period points in the minimum interval.

$$\text{PHASE SHIFT} = 0$$

$$\text{PERIOD} = \frac{2\pi}{2} = \pi \quad \frac{1}{4} \text{ PERIOD} = \frac{\pi}{4}$$

$\theta$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$r$	3	1	-1

- [h] Plot the important points and graph the function.

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