

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT

Use the power reducing formulae to rewrite $\sin^4 x$ in terms of the first powers of cosines.

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$$\begin{aligned}\sin^4 x &= (\sin^2 x)^2 \\ &= \left(\frac{1 - \cos 2x}{2} \right)^2 \\ &= \frac{1 - 2\cos 2x + \cos^2 2x}{4} \\ &= \frac{1 - 2\cos 2x + \frac{1 + \cos 4x}{2}}{4} = \frac{3 - 4\cos 2x + \cos 4x}{8}\end{aligned}$$

Graph the conic with polar equation $r = \frac{5}{2 - 3\cos\theta}$ by answering the following questions first.

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You must use techniques discussed either in lecture or in the handouts.

- [a] Determine the eccentricity and type of the conic.

$$r = \frac{\frac{5}{2}}{1 - \frac{3}{2}\cos\theta} \quad e = \frac{3}{2} \Rightarrow \text{HYPERBOLA}$$

$\frac{1}{2}$ POINT EACH

- [b] Determine the equation of the directrix.

$$ep = \frac{5}{2} \Rightarrow \frac{3}{2}p = \frac{5}{2} \Rightarrow p = \frac{5}{3} \\ x = -\frac{5}{3}$$

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- [c] Determine the rectangular co-ordinates of the x - and y -intercepts.

$$\begin{array}{cccccc} \theta & 0 & \frac{\pi}{2} & \pi & \frac{3\pi}{2} & \\ r & -5 & \frac{5}{2} & 1 & \frac{5}{2} & \end{array} \quad \begin{array}{cc} (-5, 0) & (-1, 0) \\ (0, \frac{5}{2}) & (0, -\frac{5}{2}) \end{array}$$

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- [d] Determine the rectangular co-ordinates of the vertices.

$$(-5, 0) \quad (-1, 0)$$

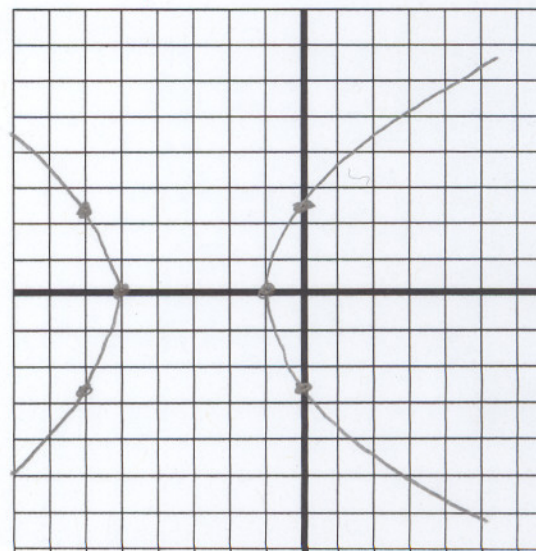
- [e] Determine the rectangular co-ordinates of both foci.

$$\text{CENTER} = \left(\frac{-5 + -1}{2}, \frac{0 + 0}{2} \right) = (-3, 0) \\ \text{FOCUS} = (-6, 0) \text{ AND } (0, 0)$$

- [f] Determine the rectangular co-ordinates of the endpoints of both latera recta.

$$(0, \frac{5}{2}) \quad (0, -\frac{5}{2}) \quad (-6, \frac{5}{2}) \quad (-6, -\frac{5}{2})$$

- [g] Plot the points you found which lie on the conic, and then sketch the conic.



Graph the polar function $r = 1 - 2\cos 2\theta$ by answering the following questions first.
You must use techniques discussed either in lecture or in the handouts.

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- [a] Determine if the graph is symmetric over the polar axis.

$$\begin{aligned} r &= 1 - 2\cos 2(-\theta) \\ r &= 1 - 2\cos 2\theta \quad \frac{1}{2} \\ \text{SYMMETRIC} \end{aligned}$$

- [b] Determine if the graph is symmetric about the pole.

$$\begin{aligned} r &= 1 - 2\cos 2(\pi + \theta) \\ r &= 1 - 2\cos (2\pi + 2\theta) \quad \frac{1}{2} \\ r &= 1 - 2[\cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta] \\ \frac{1}{2} r &= 1 - 2\cos 2\theta \quad \text{SYMMETRIC} \end{aligned}$$

- [c] Determine if the graph is symmetric over $\theta = \frac{\pi}{2}$.

SINCE THE GRAPH IS SYMMETRIC OVER BOTH THE POLAR AXIS AND THE POLE, IT IS AUTOMATICALLY SYMMETRIC OVER $\theta = \frac{\pi}{2}$.

- [d] What is the minimum interval of the graph that must be plotted first?

$$\left[0, \frac{\pi}{2}\right]$$

- [e] Find all values of θ within the minimum interval at which the graph passes through the pole.

$$\begin{aligned} 1 - 2\cos 2\theta &= 0 \quad \text{FOR } 0 \leq \theta \leq \frac{\pi}{2} \Rightarrow 0 \leq 2\theta \leq \pi \\ \cos 2\theta &= \frac{1}{2} \quad \frac{1}{2} \\ 2\theta &= \frac{\pi}{3} \\ \theta &= \frac{\pi}{6} \quad \frac{1}{2} \end{aligned}$$

- [f] Find the maximum and minimum values of r .

$$\begin{aligned} \text{AMPLITUDE} &= 2 & \text{MAX} &= 3 \quad \frac{1}{2} \\ \text{MIDLINE} &= 1 & \text{MIN} &= -1 \quad \frac{1}{2} \end{aligned}$$

- [g] Find r and θ for all the quarter period points in the minimum interval.

$$\begin{aligned} \text{PHASE SHIFT} &= 0 \\ \text{PERIOD} &= \frac{2\pi}{2} = \pi \quad \frac{1}{2} \quad \frac{1}{4} \text{ PERIOD} = \frac{\pi}{4} \\ \theta & \quad \boxed{0} \quad \boxed{\frac{\pi}{4}} \quad \boxed{\frac{\pi}{2}} \\ r & \quad \boxed{-1} \quad \boxed{1} \quad \boxed{3} \end{aligned}$$

- [h] Plot the important points and graph the function.

