Math 49B (8:30am – 9:20am)
Quiz 6 Version A
Thu Feb 17, 2011

What month is your birthday ? __ __
What are the first 2 digits of your address ? __ __
What are the last 2 digits of your zip code ? __ __
What are the last 2 digits of your DeAnza ID number ? __ __

SCORE: ___ / 30 POINTS

NO CALCULATORS ALLOWED

YOU MUST SHOW APPROPRIATE WORK TO RECEIVE FULL CREDIT

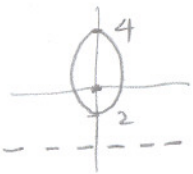
Find the polar equations of the following conics. **SCORE: ___ / 7 POINTS**

[a] parabola with focus at the pole and directrix $y = -4$

$$r = \frac{ep}{1 - e\sin\theta} = \frac{1(4)}{1-(1)\sin\theta} = \frac{\boxed{4}^{\frac{1}{2}}}{\underbrace{1}_{\frac{1}{2}} \underbrace{-}_{\frac{1}{2}} \underbrace{\sin\theta}_{\frac{1}{2}}}$$

← COEFFICIENT OF $\sin\theta$ MUST BE 1

[b] ellipse with focus at the pole and vertices at $\left(4, \frac{\pi}{2}\right)$ and $\left(2, \frac{3\pi}{2}\right)$.



1 $\boxed{r = \frac{ep}{1 - e\sin\theta}} = \boxed{\frac{\frac{1}{3}(8)}{1-\frac{1}{3}\sin\theta}}^{\frac{1}{2}} = \boxed{\frac{8}{3-\sin\theta}}^{\frac{1}{2}}$

$\frac{1}{2}$ $\boxed{4 = \frac{ep}{1-e\sin\frac{\pi}{2}}}$ $\quad$ $\boxed{2 = \frac{ep}{1-e\sin\frac{3\pi}{2}}}$ $\frac{1}{2}$

$4 = \frac{ep}{1-e}$ $\quad$ $2 = \frac{ep}{1+e}$

$ep = 4 - 4e$ $\quad$ $ep = 2 + 2e$

$4 - 4e = 2 + 2e$

$2 = 6e$

1 $\boxed{e = \frac{1}{3}}$

$\frac{1}{3}p = 4 - 4\left(\frac{1}{3}\right)$

$\frac{1}{3}p = \frac{8}{3}$

1 $\boxed{p = 8}$

Find parametric equations for the circle with center $(4, -2)$ and radius 9. **SCORE: ___ / 2 POINTS**

$\boxed{x = 4 + 9\cos t}$, 1

$\boxed{y = -2 + 9\sin t}$, 1

Consider the curve represented by the parametric equations $\begin{array}{l} x = 4 - 2t \\ y = \dfrac{10t}{t+2} \end{array}$. **SCORE: ___ / 4 POINTS**

Find the corresponding rectangular equation by eliminating the parameter. **SIMPLIFY YOUR FINAL ANSWER.**

$x = 4 - 2t$

$2t = 4 - x$

$\underbrace{t = \frac{4-x}{2}}_{1\frac{1}{2}}$

$\underbrace{y = \frac{10\left(\frac{4-x}{2}\right)}{\frac{4-x}{2} + 2}}_{1\frac{1}{2}} = \frac{10(4-x)}{4-x+4} = \underbrace{\frac{40-10x}{8-x}}_{1}$

The initial point of vector $\vec{u}$ is $(-5, -1)$, and the terminal point is $(-9, 2)$. **SCORE: ___ / 11 POINTS**

The component form of vector $\vec{w}$ is $< -2, -3 >$.

[a] Write $\vec{u}$ as a linear combination of the standard unit vectors $\vec{i}$ and $\vec{j}$.

$$\vec{u} = \langle -9 - -5, 2 - -1 \rangle$$
$$= \langle -4, 3 \rangle$$
$$= -4\vec{i} + 3\vec{j}$$

1 1

[b] Find a unit vector in the same direction as $\vec{u}$.

$$\frac{1}{\|\vec{u}\|}\vec{u} = \frac{1}{\sqrt{(-4)^2 + 3^2}} \langle -4, 3 \rangle = \frac{1}{5} \langle -4, 3 \rangle = \left\langle -\frac{4}{5}, \frac{3}{5} \right\rangle$$

2 1 1

[c] Find the component form of $2\vec{u} - 3\vec{w}$.

$$2\langle -4, 3 \rangle - 3\langle -2, -3 \rangle$$
$$= \langle -8, 6 \rangle - \langle -6, -9 \rangle$$
$$= \langle -2, 15 \rangle$$

1 1

[d] Determine if $\vec{u}$ and $\vec{w}$ are orthogonal.

$$\langle -4, 3 \rangle \cdot \langle -2, -3 \rangle \quad 1$$
$$= (-4)(-2) + 3(-3)$$
$$= -1 \neq 0 \quad 1$$

NOT ORTHOGONAL 1

The magnitude of vector $\vec{u}$ is 4. The magnitude of vector $\vec{w}$ is 5. The angle between the two vectors is 60°. **SCORE: ___ / 2 POINTS**

Find $\vec{u} \cdot \vec{w}$.

$$\|\vec{u}\| \, \|\vec{w}\| \cos\theta = 4 \cdot 5 \cdot \cos 60^\circ = 10$$

1 1

Find the angle between the vectors $< -3, 1 >$ and $< 2, 1 >$. **SCORE: ___ / 4 POINTS**

$$\theta = \cos^{-1} \frac{\langle -3, 1 \rangle \cdot \langle 2, 1 \rangle}{\|\langle -3, 1 \rangle\| \, \|\langle 2, 1 \rangle\|}$$

$$= \cos^{-1} \frac{-5}{\sqrt{10}\sqrt{5}}$$

1 ½ ½

$$= \cos^{-1} \frac{-\cancel{5}}{\cancel{5}\sqrt{2}}$$

$$= \cos^{-1} -\frac{\sqrt{2}}{2} \quad 1$$

$$= \frac{3\pi}{4} \text{ or } 135^\circ \quad 1$$

EITHER ANSWER IS ACCEPTABLE