[redacted]

Evaluate the following integrals. **SCORE: _____ / 60 PTS**

[a] $\displaystyle\int_{-2}^{2}(x^3-3x)\sqrt{1+x^4}\,dx$

$((-x)^3-3(-x))\sqrt{1+(-x)^4}$

$=(-x^3+3x)\sqrt{1+x^4}$

$=-(x^3-3x)\sqrt{1+x^4}$

INTEGRAND IS ODD + CONTINUOUS

SO INTEGRAL $=0$

[b] $\displaystyle\int_{-1}^{1}\frac{2x+1}{\sqrt{4x+5}}\,dx$

$u=\sqrt{4x+5}\rightarrow x=\frac{1}{4}(u^2-5)$

$\frac{du}{dx}=\frac{1}{2\sqrt{4x+5}}\cdot 4=\frac{2}{\sqrt{4x+5}}$

$dx=\frac{\sqrt{4x+5}}{2}du$

$\frac{2x+1}{\sqrt{4x+5}}dx=\frac{2x+1}{\sqrt{4x+5}}\frac{\sqrt{4x+5}}{2}du$

$=\frac{2(\frac{1}{4}(u^2-5))+1}{2}du$

$=\frac{\frac{1}{2}u^2-\frac{3}{2}}{2}du$

$=(\frac{1}{4}u^2-\frac{3}{4})du$

$\int_1^3(\frac{1}{4}u^2-\frac{3}{4})du$

$=(\frac{1}{12}u^3-\frac{3}{4}u)\Big|_1^3$

$=\frac{1}{12}(3^3-1^3)-\frac{3}{4}(3-1)$

$=\frac{1}{12}(26)-\frac{3}{4}(2)$

$=\frac{13}{6}-\frac{3}{2}$

$=\frac{4}{6}=\frac{2}{3}$

SEE VERSION N KEY FOR ALTERNATE SOLUTION

[c] $\displaystyle\int\frac{1}{(x^2-1)\tanh^{-1}x}\,dx$

$u=\tanh^{-1}x$

$\frac{du}{dx}=\frac{1}{1-x^2}$

$-du=\frac{1}{x^2-1}dx$

$-\int\frac{1}{u}du$

$=-\ln|u|+C$

$=-\ln|\tanh^{-1}x|+C$

[redacted]

State both parts of the Fundamental Theorem of Calculus and the Net Change Theorem. **SCORE: _____ / 10 PTS**

Use complete sentences and proper algebra & English as shown in class.

① IF f IS CONTINUOUS ON $[c,b]$ AND $a \in [c,b]$
AND $g(x) = \int_a^x f(t)\,dt$, THEN $g'(x) = f(x)$ FOR ALL $x \in [c,b]$

② IF f IS CONTINUOUS ON $[a,b]$ AND $F'(x) = f(x)$ ON $[a,b]$
THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

NCT IF F' IS CONTINUOUS ON $[a,b]$, THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

[redacted]

The table gives the rate $r(t)$ at which rainwater is falling into a bucket (in fluid ounces/minute) at various times (in minutes). At time $t = 2$, there were 6 fluid ounces of rainwater in the bucket. **SCORE: _____ / 20 PTS**

t	0	2	4	6	8	10	12	14	16	18	20	22	24	26
$r(t)$	1	3	2	0	2	1	3	4	0	2	3	1	0	4

[a] Write an expression (involving an integral) for the amount of rainwater in the bucket at $t = 26$.

IF $A(t)$ = AMOUNT OF RAINWATER IN BUCKET AT TIME t
THEN $A'(t) = r(t)$
SO $\int_2^{26} r(t)\,dt = A(26) - A(2)$
$A(26) = 6 + \int_2^{26} r(t)\,dt$

[b] Estimate the amount of rainwater in the bucket at $t = 26$ using [a], 4 subintervals and left endpoints.

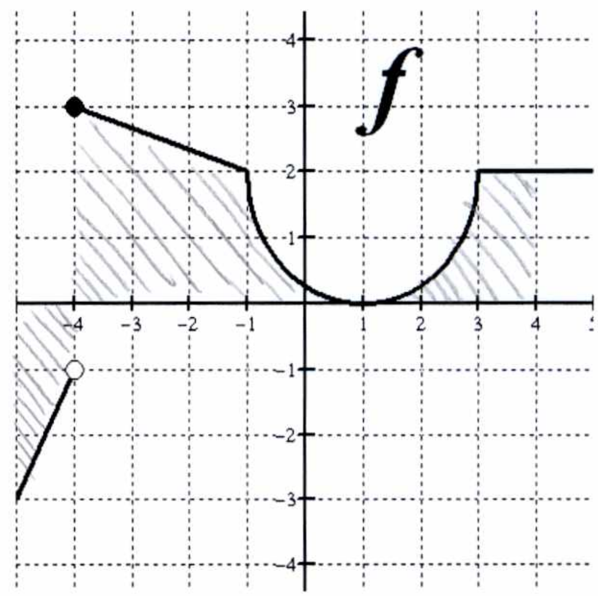
$\Delta x = \frac{26-2}{4} = 6$

2 8 14 20 26

$6 + [r(2) + r(8) + r(14) + r(20)]\,6$
$= 6 + (3+2+4+3)\,6$
$= 78$ FLUID OUNCES

The graph of f is shown on the right, and consists of 2 line segments, a semi-circle and another line segment. **SCORE: _____ / 40 PTS**

Let $g(x) = \int_4^x f(t)\, dt$.



[a] Find $g(-5)$. $= \int_4^{-5} f(t)\,dt = -\int_{-5}^{4} f(t)\,dt$

$$= -\left(-\frac{3+1}{2}\cdot 1 + \frac{3+2}{2}\cdot 3 + \left(8 - \frac{1}{2}\cdot 4\pi\right) + 2\right)$$

$$= -\left(-2 + \frac{15}{2} + 8 - 2\pi + 2\right)$$

$$= 2\pi - \frac{31}{2}$$

[b] Find the x – coordinates of all inflection points of g. **<u>Explain your answer very briefly.</u>**

$g' = f$ CHANGES FROM INCREASING TO DECREASING AT $x = -4$

DECREASING INCREASING $x = 1$

[c] If $k(x) = \int_4^{x^2+2} f(t)\, dt$, find $k'(-1)$.

$$k'(x) = \frac{d}{d(x^2+2)} \int_4^{x^2+2} f(t)\,dt \cdot \frac{d(x^2+2)}{dx}$$

$$= f(x^2+2)\cdot 2x$$

$$k'(-1) = f(3)\cdot 2(-1) = 2\cdot -2 = -4$$

Prove that $8 \le \int_0^4 (x + \cos^4 x)\, dx \le 12$. **SCORE: _____ / 20 PTS**

$0 \le \cos^4 x \le 1$ FOR ALL x

$$x \le x + \cos^4 x \le x + 1$$

$$\int_0^4 x\,dx \le \int_0^4 (x + \cos^4 x)\,dx \le \int_0^4 (x+1)\,dx$$

$$\frac{1}{2}x^2\Big|_0^4 \le \int_0^4 (x + \cos^4 x)\,dx \le \left(\frac{1}{2}x^2 + x\right)\Big|_0^4$$

$$\frac{1}{2}(4^2 - 0^2) = 8 \le \int_0^4 (x + \cos^4 x)\,dx \le 12 = \frac{1}{2}(4^2 - 0^2) + (4 - 0)$$