

State both parts of the Fundamental Theorem of Calculus and the Net Change Theorem.

SCORE: ____ / 10 PTS

Use complete sentences and proper algebra & English as shown in class.

① IF f IS CONTINUOUS ON $[c, b]$ AND $a \in [c, b]$
AND $g(x) = \int_a^x f(t) dt$, THEN $g'(x) = f(x)$ FOR ALL $x \in [c, b]$

② IF f IS CONTINUOUS ON $[a, b]$ AND $F'(x) = f(x)$ ON $[a, b]$
THEN $\int_a^b f(x) dx = F(b) - F(a)$

(NCT) IF F' IS CONTINUOUS ON $[a, b]$, THEN $\int_a^b F'(x) dx = F(b) - F(a)$

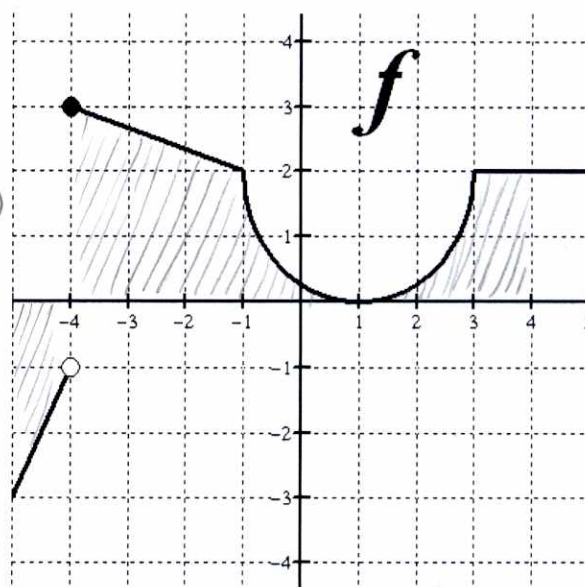
The graph of f is shown on the right, and consists of 2 line segments, a semi-circle and another line segment.

SCORE: ____ / 40 PTS

Let $g(x) = \int_4^x f(t) dt$.

[a] Find $g(-5)$.

$$\begin{aligned} g(-5) &= \int_4^{-5} f(t) dt = -\int_{-5}^4 f(t) dt \\ &= -\left(-\frac{3+1}{2} \cdot 1 + \frac{3+2}{2} \cdot 3 + \left(8 - \frac{1}{2} \cdot 4\pi\right) + 2\right) \\ &= -\left(-2 + \frac{15}{2} + 8 - 2\pi + 2\right) \\ &= 2\pi - \frac{31}{2} \end{aligned}$$



[b] Find the x -coordinates of all inflection points of g . Explain your answer very briefly.

$g' = f$ CHANGES FROM INCREASING TO DECREASING AT $x = -4$
DECREASING INCREASING $x = 1$

[c] If $k(x) = \int_4^{x^2-1} f(t) dt$, find $k'(-2)$.

$$\begin{aligned} k'(x) &= \frac{d}{d(x^2-1)} \int_4^{x^2-1} f(t) dt \cdot \frac{d(x^2-1)}{dx} \\ &= f(x^2-1) \cdot 2x \end{aligned}$$

$$k'(-2) = f(3) \cdot 2(-2) = 2 \cdot -4 = -8$$

Evaluate the following integrals.

SCORE: ____ / 60 PTS

[a] $\int_{-2}^2 (x^5 + 2x)\sqrt{1+x^2} dx$

$$((-x)^5 + 2(-x))\sqrt{1+(-x)^2}$$

$$= (-x^5 - 2x)\sqrt{1+x^2}$$

$$= -(x^5 + 2x)\sqrt{1+x^2}$$

INTEGRAND IS ODD + CONTINUOUS

SO INTEGRAL = 0

[b] $\int_{-1}^1 \frac{2x+1}{\sqrt{4x+5}} dx$

$$u = 4x+5 \rightarrow x = \frac{1}{4}(u-5)$$

$$\frac{du}{dx} = 4$$

$$dx = \frac{1}{4} du$$

$$\frac{2x+1}{\sqrt{4x+5}} dx = \frac{1}{4} \frac{2x+1}{\sqrt{4x+5}} du$$

$$= \frac{1}{4} \frac{2(\frac{1}{4}(u-5))+1}{\sqrt{u}} du$$

$$= \frac{\frac{1}{2}u - \frac{3}{2}}{4\sqrt{u}} du$$

$$= (\frac{1}{8}u^{\frac{1}{2}} - \frac{3}{8}u^{-\frac{1}{2}}) du$$

$$\int_1^9 (\frac{1}{8}u^{\frac{1}{2}} - \frac{3}{8}u^{-\frac{1}{2}}) du$$

$$= (\frac{1}{8} \cdot \frac{2}{3} u^{\frac{3}{2}} - \frac{3}{8} \cdot 2u^{\frac{1}{2}}) \Big|_1^9$$

$$= \frac{1}{12} (9^{\frac{3}{2}} - 1^{\frac{3}{2}}) - \frac{3}{4} (9^{\frac{1}{2}} - 1^{\frac{1}{2}})$$

$$= \frac{1}{12} (26) - \frac{3}{4} (2)$$

$$= \frac{13}{6} - \frac{3}{2}$$

$$= \frac{4}{6} = \frac{2}{3}$$

[c] $\int \frac{1}{(x^2-1)\tanh^{-1}x} dx$

$$u = \tanh^{-1}x$$

$$\frac{du}{dx} = \frac{1}{1-x^2}$$

$$-du = \frac{1}{x^2-1} dx$$

$$-\int \frac{1}{u} du$$

$$= -\ln|u| + C$$

$$= -\ln|\tanh^{-1}x| + C$$

SEE VERSION I KEY
FOR ALTERNATE
SOLUTION

Prove that $8 \leq \int_0^4 (x + \cos^4 x) dx \leq 12$.

SCORE: ____ / 20 PTS

$$0 \leq \cos^4 x \leq 1 \quad \text{FOR ALL } x$$

$$x \leq x + \cos^4 x \leq x + 1$$

$$\int_0^4 x dx \leq \int_0^4 (x + \cos^4 x) dx \leq \int_0^4 (x + 1) dx$$

$$\frac{1}{2}x^2 \Big|_0^4 \leq \int_0^4 (x + \cos^4 x) dx \leq \left(\frac{1}{2}x^2 + x\right) \Big|_0^4$$

$$\frac{1}{2}(4^2 - 0^2) = 8 \leq \int_0^4 (x + \cos^4 x) dx \leq 12 = \frac{1}{2}(4^2 - 0^2) + (4 - 0)$$

The table gives the rate $r(t)$ at which rainwater is falling into a bucket (in fluid ounces/minute) at various times (in minutes). At time $t = 2$, there were 7 fluid ounces of rainwater in the bucket.

SCORE: ____ / 20 PTS

t	0	2	4	6	8	10	12	14	16	18	20	22	24	26
$r(t)$	1	3	2	0	2	1	3	4	0	2	3	1	0	4

[a] Write an expression (involving an integral) for the amount of rainwater in the bucket at $t = 26$.

IF $A(t)$ = AMOUNT OF RAINWATER IN BUCKET AT TIME t

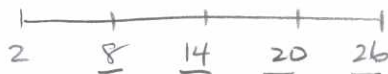
THEN $A'(t) = r(t)$

$$\text{SO } \int_2^{26} r(t) dt = A(26) - A(2)$$

$$\begin{aligned} A(26) &= A(2) + \int_2^{26} r(t) dt \\ &= 7 + \int_2^{26} r(t) dt \end{aligned}$$

[b] Estimate the amount of rainwater in the bucket at $t = 26$ using [a], 4 subintervals and right endpoints.

$$\Delta x = \frac{26-2}{4} = 6$$



$$7 + [r(8) + r(14) + r(20) + r(26)] 6$$

$$= 7 + (2 + 4 + 3 + 4) 6$$

$$= 85 \text{ FLUID OUNCES}$$