
Prove that $8 \le \int_0^4 (x + \cos^4 x)\, dx \le 12$.

SCORE: _____ / 20 PTS

$0 \le \cos^4 x \le 1$ FOR ALL X

$x \le x + \cos^4 x \le x + 1$

$\int_0^4 x\, dx \le \int_0^4 (x + \cos^4 x)\, dx \le \int_0^4 (x+1)\, dx$

$\frac{1}{2}x^2 \Big|_0^4 \le \int_0^4 (x + \cos^4 x)\, dx \le \left(\frac{1}{2}x^2 + x\right)\Big|_0^4$

$\frac{1}{2}(4^2 - 0^2) = 8 \le \int_0^4 (x + \cos^4 x)\, dx \le 12 = \frac{1}{2}(4^2 - 0^2) + (4 - 0)$

The table gives the rate $r(t)$ at which rainwater is falling into a bucket (in fluid ounces/minute) at various times (in minutes). At time $t = 2$, there were 7 fluid ounces of rainwater in the bucket.

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t	0	2	4	6	8	10	12	14	16	18	20	22	24	26
$r(t)$	1	3	2	0	2	1	3	4	0	2	3	1	0	4

[a] Write an expression (involving an integral) for the amount of rainwater in the bucket at $t = 26$.

IF $A(t)$ = AMOUNT OF RAINWATER IN BUCKET AT TIME t

THEN $A'(t) = r(t)$

SO $\int_2^{26} r(t)\, dt = A(26) - A(2)$

$A(26) = A(2) + \int_2^{26} r(t)\, dt$

$= 7 + \int_2^{26} r(t)\, dt$

[b] Estimate the amount of rainwater in the bucket at $t = 26$ using [a], 4 subintervals and right endpoints.

$\Delta x = \frac{26-2}{4} = 6$

2 $\underline{8}$ $\underline{14}$ $\underline{20}$ $\underline{26}$

$7 + [r(8) + r(14) + r(20) + r(26)]6$

$= 7 + (2 + 4 + 3 + 4)6$

$= 85$ FLUID OUNCES

[redacted]

Evaluate the following integrals.

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[a] $\displaystyle\int_{-2}^{2}(x^5+2x)\sqrt{1+x^2}\,dx$

$((-x)^5+2(-x))\sqrt{1+(-x)^2}$

$=(-x^5-2x)\sqrt{1+x^2}$

$=-(x^5+2x)\sqrt{1+x^2}$

INTEGRAND IS ODD + CONTINUOUS

SO INTEGRAL = 0

[b] $\displaystyle\int_{-1}^{1}\frac{2x+1}{\sqrt{4x+5}}\,dx$

$u=4x+5 \longrightarrow x=\frac{1}{4}(u-5)$

$\frac{du}{dx}=4$

$dx=\frac{1}{4}du$

$\frac{2x+1}{\sqrt{4x+5}}dx=\frac{1}{4}\frac{2x+1}{\sqrt{4x+5}}du$

$=\frac{1}{4}\frac{2(\frac{1}{4}(u-5))+1}{\sqrt{u}}du$

$=\frac{\frac{1}{2}u-\frac{3}{2}}{4\sqrt{u}}du$

$=(\frac{1}{8}u^{\frac{1}{2}}-\frac{3}{8}u^{-\frac{1}{2}})du$

$\int_1^9(\frac{1}{8}u^{\frac{1}{2}}-\frac{3}{8}u^{-\frac{1}{2}})du$

$=(\frac{1}{8}\cdot\frac{2}{3}u^{\frac{3}{2}}-\frac{3}{8}\cdot 2u^{\frac{1}{2}})\Big|_1^9$

$=\frac{1}{12}(9^{\frac{3}{2}}-1^{\frac{3}{2}})-\frac{3}{4}(9^{\frac{1}{2}}-1^{\frac{1}{2}})$

$=\frac{1}{12}(26)-\frac{3}{4}(2)$

$=\frac{13}{6}-\frac{3}{2}$

$=\frac{4}{6}=\frac{2}{3}$

SEE VERSION I KEY FOR ALTERNATE SOLUTION

[c] $\displaystyle\int\frac{1}{(x^2-1)\tanh^{-1}x}\,dx$

$u=\tanh^{-1}x$

$\frac{du}{dx}=\frac{1}{1-x^2}$

$-du=\frac{1}{x^2-1}dx$

$-\int\frac{1}{u}du$

$=-\ln|u|+C$

$=-\ln|\tanh^{-1}x|+C$

[redacted]

State both parts of the Fundamental Theorem of Calculus and the Net Change Theorem. **SCORE: _____ / 10 PTS**

Use complete sentences and proper algebra & English as shown in class.

① IF f IS CONTINUOUS ON $[c,b]$ AND $a \in [c,b]$ AND $g(x) = \int_a^x f(t)\,dt$, THEN $g'(x) = f(x)$ FOR ALL $x \in [c,b]$

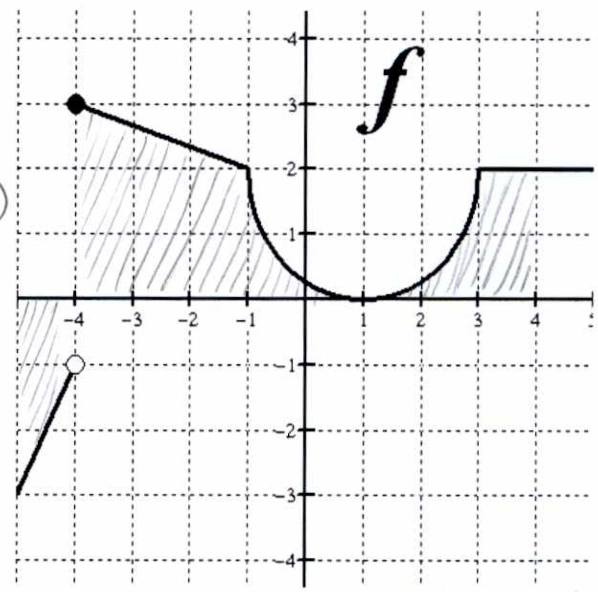
② IF f IS CONTINUOUS ON $[a,b]$ AND $F'(x) = f(x)$ ON $[a,b]$ THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

NCT IF F' IS CONTINUOUS ON $[a,b]$, THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

[redacted]

The graph of f is shown on the right, and consists of 2 line segments, a semi-circle and another line segment. **SCORE: _____ / 40 PTS**

Let $g(x) = \int_4^x f(t)\,dt$.



[a] Find $g(-5)$. $= \int_4^{-5} f(t)\,dt = -\int_{-5}^{4} f(t)\,dt$

$= -\left(-\frac{3+1}{2}\cdot 1 + \frac{3+2}{2}\cdot 3 + (8 - \frac{1}{2}\cdot 4\pi) + 2\right)$

$= -\left(-2 + \frac{15}{2} + 8 - 2\pi + 2\right)$

$= 2\pi - \frac{31}{2}$

[b] Find the $x-$coordinates of all inflection points of g. **Explain your answer very briefly.**

$g' = f$ CHANGES FROM INCREASING TO DECREASING AT $x = -4$

DECREASING INCREASING $x = 1$

[c] If $k(x) = \int_4^{x^2-1} f(t)\,dt$, find $k'(-2)$.

$k'(x) = \frac{d}{d(x^2-1)} \int_4^{x^2-1} f(t)\,dt \cdot \frac{d(x^2-1)}{dx}$

$= f(x^2-1)\cdot 2x$

$k'(-2) = f(3)\cdot 2(-2) = 2\cdot -4 = -8$