[redacted]

Evaluate the following integrals. **SCORE: _____ / 60 PTS**

[a] $\int_{-2}^{2} (x^3 - 3x)\sqrt{1+x^4}\, dx$

$((-x)^3 - 3(-x))\sqrt{1+(-x)^4}$

$= (-x^3 + 3x)\sqrt{1+x^4}$

$= -(x^3 - 3x)\sqrt{1+x^4}$

INTEGRAND IS ODD + CONTINUOUS

SO INTEGRAL $= 0$

[b] $\int_{-1}^{1} \frac{2x+1}{\sqrt{4x+5}}\, dx$

$u = \sqrt{4x+5} \rightarrow x = \frac{1}{4}(u^2 - 5)$

$\frac{du}{dx} = \frac{1}{2\sqrt{4x+5}} \cdot 4 = \frac{2}{\sqrt{4x+5}}$

$dx = \frac{\sqrt{4x+5}}{2}\, du$

$\frac{2x+1}{\sqrt{4x+5}}\, dx = \frac{2x+1}{\sqrt{4x+5}} \frac{\sqrt{4x+5}}{2}\, du$

$= \frac{2(\frac{1}{4}(u^2-5))+1}{2}\, du$

$= \frac{\frac{1}{2}u^2 - \frac{3}{2}}{2}\, du$

$= (\frac{1}{4}u^2 - \frac{3}{4})\, du$

$\int_1^3 (\frac{1}{4}u^2 - \frac{3}{4})\, du$

$= (\frac{1}{12}u^3 - \frac{3}{4}u)\Big|_1^3$

$= \frac{1}{12}(3^3 - 1^3) - \frac{3}{4}(3-1)$

$= \frac{1}{12}(26) - \frac{3}{4}(2)$

$= \frac{13}{6} - \frac{3}{2}$

$= \frac{4}{6} = \frac{2}{3}$

SEE VERSION N KEY FOR ALTERNATE SOLUTION

[c] $\int \frac{1}{(x^2-1)\tanh^{-1}x}\, dx$

$u = \tanh^{-1}x$

$\frac{du}{dx} = \frac{1}{1-x^2}$

$-du = \frac{1}{x^2-1}\, dx$

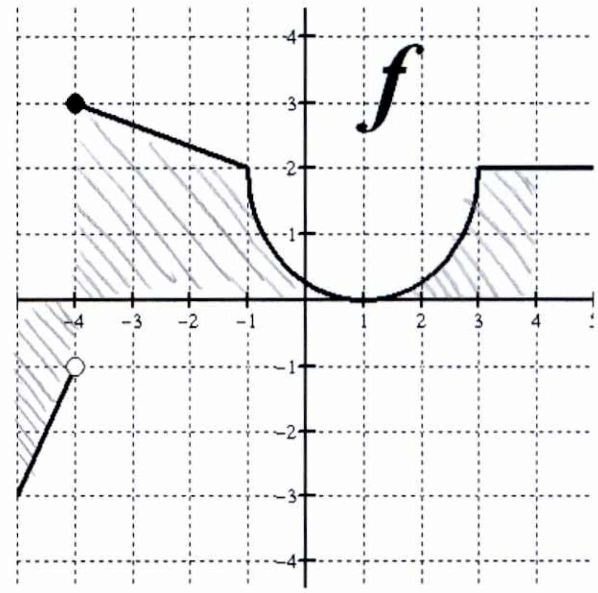
$-\int \frac{1}{u}\, du$

$= -\ln|u| + C$

$= -\ln|\tanh^{-1}x| + C$

The graph of f is shown on the right, and consists of 2 line segments, a semi-circle and another line segment. **SCORE: _____ / 40 PTS**

Let $g(x) = \int_4^x f(t)\,dt$.



[a] Find $g(-5)$. $= \int_4^{-5} f(t)\,dt = -\int_{-5}^{4} f(t)\,dt$

$= -\left(-\frac{3+1}{2}\cdot 1 + \frac{3+2}{2}\cdot 3 + \left(8 - \frac{1}{2}\cdot 4\pi\right) + 2\right)$

$= -\left(-2 + \frac{15}{2} + 8 - 2\pi + 2\right)$

$= 2\pi - \frac{31}{2}$

[b] Find the x – coordinates of all inflection points of g. **<u>Explain your answer very briefly.</u>**

$g' = f$ CHANGES FROM INCREASING TO DECREASING AT $x = -4$

DECREASING INCREASING $x = 1$

[c] If $k(x) = \int_4^{x^2+2} f(t)\,dt$, find $k'(-1)$.

$k'(x) = \frac{d}{d(x^2+2)} \int_4^{x^2+2} f(t)\,dt \cdot \frac{d(x^2+2)}{dx}$

$= f(x^2+2)\cdot 2x$

$k'(-1) = f(3)\cdot 2(-1) = 2\cdot -2 = -4$

Prove that $8 \le \int_0^4 (x + \cos^4 x)\,dx \le 12$. **SCORE: _____ / 20 PTS**

$0 \le \cos^4 x \le 1$ FOR ALL x

$x \le x + \cos^4 x \le x + 1$

$\int_0^4 x\,dx \le \int_0^4 (x + \cos^4 x)\,dx \le \int_0^4 (x+1)\,dx$

$\frac{1}{2}x^2 \Big|_0^4 \le \int_0^4 (x + \cos^4 x)\,dx \le \left(\frac{1}{2}x^2 + x\right)\Big|_0^4$

$\frac{1}{2}(4^2 - 0^2) = 8 \le \int_0^4 (x + \cos^4 x)\,dx \le 12 = \frac{1}{2}(4^2 - 0^2) + (4 - 0)$

State both parts of the Fundamental Theorem of Calculus and the Net Change Theorem. **SCORE: _____ / 10 PTS**
<u>Use complete sentences and proper algebra & English as shown in class.</u>

① IF f IS CONTINUOUS ON $[c,b]$ AND $a \in [c,b]$
AND $g(x) = \int_a^x f(t)\,dt$, THEN $g'(x) = f(x)$ FOR ALL $x \in [c,b]$

② IF f IS CONTINUOUS ON $[a,b]$ AND $F'(x) = f(x)$ ON $[a,b]$
THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

NCT IF F' IS CONTINUOUS ON $[a,b]$, THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

The table gives the rate $r(t)$ at which rainwater is falling into a bucket (in fluid ounces/minute) at various times (in minutes). At time $t = 2$, there were 6 fluid ounces of rainwater in the bucket. **SCORE: _____ / 20 PTS**

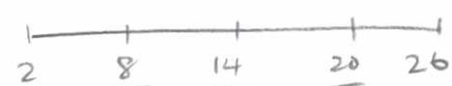
t	0	2	4	6	8	10	12	14	16	18	20	22	24	26
$r(t)$	1	3	2	0	2	1	3	4	0	2	3	1	0	4

[a] Write an expression (involving an integral) for the amount of rainwater in the bucket at $t = 26$.

IF $A(t)$ = AMOUNT OF RAINWATER IN BUCKET AT TIME t
THEN $A'(t) = r(t)$
SO $\int_2^{26} r(t)\,dt = A(26) - A(2)$
$A(26) = 6 + \int_2^{26} r(t)\,dt$

[b] Estimate the amount of rainwater in the bucket at $t = 26$ using [a], 4 subintervals and left endpoints.

$\Delta x = \frac{26-2}{4} = 6$



$6 + [r(2) + r(8) + r(14) + r(20)]6$
$= 6 + (3 + 2 + 4 + 3)6$
$= 78$ FLUID OUNCES