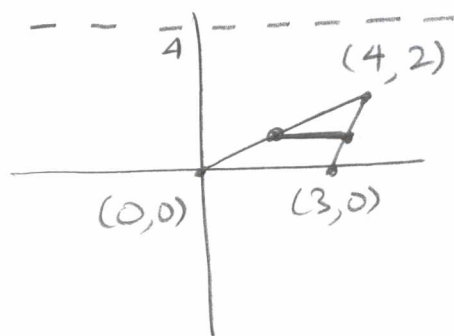


The region defined by $y \leq \frac{1}{2}x$, $y \geq 2x - 6$ and $y \geq 0$ is revolved around the line $y = 4$.

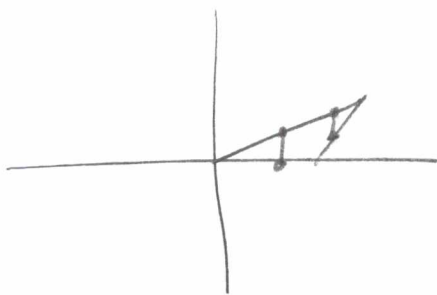
SCORE: ____ / 40 PTS

- [a] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the shell method.



$$2\pi \int_0^2 (4-y) \left(\frac{y+6}{2} - 2y \right) dy$$

- [b] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the disk or washer method.



$$\pi \int_0^3 (4^2 - (4 - \frac{1}{2}x)^2) dx + \pi \int_3^4 ((4 - (2x - 6))^2 - (4 - \frac{1}{2}x)^2) dx$$

- [c] Find the volume of the solid using one of your integrals above. **DO NOT USE GEOMETRY NOR THE THEOREM OF PAPPUS.**

$$\begin{aligned} & 2\pi \int_0^2 (4-y) \left(3 - \frac{3}{2}y \right) dy \\ &= 2\pi \int_0^2 (12 - 9y + \frac{3}{2}y^2) dy \\ &= 2\pi \left(12y - \frac{9}{2}y^2 + \frac{1}{2}y^3 \right) \Big|_0^2 \\ &= 2\pi (24 - 18 + 4) \\ &= 20\pi \end{aligned}$$

Find the centroid of the region bounded by $y = x^2$ and $y = 2x$.

SCORE: ____ / 25 PTS

$$x^2 = 2x \rightarrow x = 0, 2$$

$$\int_0^2 (2x - x^2) dx = (x^2 - \frac{1}{3}x^3) \Big|_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

$$\int_0^2 x(2x - x^2) dx = \int_0^2 (2x^2 - x^3) dx = (\frac{2}{3}x^3 - \frac{1}{4}x^4) \Big|_0^2 = \frac{16}{3} - 4 = \frac{4}{3}$$

$$\begin{aligned} \frac{1}{2} \int_0^2 ((2x)^2 - (x^2)^2) dx &= \frac{1}{2} \int_0^2 (4x^2 - x^4) dx = \frac{1}{2} (\frac{4}{3}x^3 - \frac{1}{5}x^5) \Big|_0^2 \\ &= \frac{1}{2} (\frac{32}{3} - \frac{32}{5}) = \frac{1}{2} \cdot \frac{64}{15} \\ &= \frac{32}{15} \end{aligned}$$

$$(\bar{x}, \bar{y}) = \left(\frac{\frac{4}{3}}{\frac{4}{3}}, \frac{\frac{32}{15}}{\frac{4}{3}} \right) = \left(1, \frac{8}{5} \right)$$

Find the length of the parametric curve $x = 1 + 3t + 2e^{-6t}$
 $y = 5 - 4e^{-3t}$ over $1 \leq t \leq 3$.

SCORE: ____ / 25 PTS

$$\begin{aligned} & \int_1^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\ &= \int_1^3 \sqrt{(3 - 12e^{-6t})^2 + (12e^{-3t})^2} dt \\ &= \int_1^3 \sqrt{9 - 72e^{-6t} + 144e^{-12t} + 144e^{-6t}} dt \\ &= \int_1^3 \sqrt{9 + 72e^{-6t} + 144e^{-12t}} dt \\ &= \int_1^3 \sqrt{(3 + 12e^{-6t})^2} dt \\ &= \int_1^3 (3 + 12e^{-6t}) dt \\ &= (3t - 2e^{-6t}) \Big|_1^3 \end{aligned}$$

$\begin{aligned} &= 3(3 - 1) - 2(e^{-18} - e^{-6}) \\ &= 6 - 2e^{-18} + 2e^{-6} \end{aligned}$

The snooze button on an alarm clock allows you to turn off a ringing alarm for a short time (the “snooze period”). **SCORE: ____ / 35 PTS**
 At the end of the snooze period, the alarm rings again. To prevent you from getting used to a certain snooze period, Big Ben alarm clocks are designed so that the snooze period is a random time between 3 and 8 minutes.

Let X be a continuous random variable representing the length of a snooze period, and suppose that X has the probability density function

$$f(x) = \begin{cases} \frac{a}{\sqrt{1+x}}, & 3 \leq x \leq 8 \\ 0, & x < 3 \text{ or } x > 8 \end{cases} \quad \text{for some constant } a.$$

[a] Find a .

$$\begin{aligned} \int_3^8 a(1+x)^{-\frac{1}{2}} dx &= 1 \\ u = 1+x &\rightarrow du = dx \\ \int_4^9 a u^{-\frac{1}{2}} du &= 1 \\ 2a u^{\frac{1}{2}} \Big|_4^9 &= 1 \\ 2a(3-2) &= 1 \end{aligned} \quad \begin{aligned} 2a &= 1 \\ a &= \frac{1}{2} \end{aligned}$$

[b] Find the mean length of the snooze period.

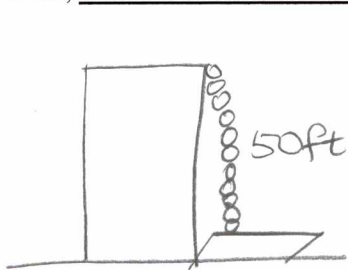
$$\begin{aligned} \int_3^8 \frac{1}{2} x (1+x)^{-\frac{1}{2}} dx &= \frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right) \Big|_4^9 \\ u = 1+x &\rightarrow du = dx \\ x &= u-1 \\ &= \frac{1}{3} (27-8) - (3-2) \\ &= \frac{19}{3} - 1 \\ &= \frac{16}{3} \end{aligned}$$

A 50 foot long chain weighing 200 pounds hangs from the roof of a 50 foot tall building.

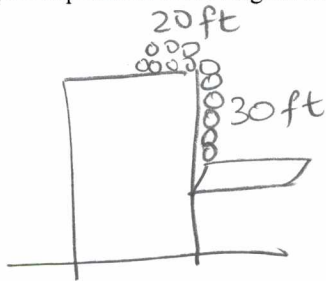
SCORE: ____ / 25 PTS

The chain is used to pull a 70 pound tabletop from ground level to a window 20 feet above ground level.

Write, **BUT DO NOT EVALUATE**, an expression involving an integral (or sum of integrals) for the work done.



BEFORE



AFTER

$$\text{DENSITY} = \frac{200 \text{ lb}}{50 \text{ ft}} = 4 \frac{\text{lb}}{\text{ft}}$$

TOP 20 FT MOVED TO ROOF
BOTTOM 30 FT + TABLETOP
MOVED 20 FT

$$\int_0^{20} 4x \, dx + (4 \cdot 30 + 70) \cdot 20$$
$$= \int_0^{20} 4x \, dx + 190 \cdot 20 \text{ ft-lb}$$