Evaluate $\displaystyle\int_1^\infty \frac{1}{x(\ln x)^3}\,dx$.

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$$= \int_1^e \frac{1}{x(\ln x)^3}\,dx + \int_e^\infty \frac{1}{x(\ln x)^3}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{u^3}\,du = -\frac{1}{2}u^{-2}$

$= -\frac{1}{2}\frac{1}{(\ln x)^2}$

$$\hookrightarrow = \lim_{N\to 1^+} \int_N^e \frac{1}{x(\ln x)^3}\,dx$$

$$= \lim_{N\to 1^+} \left(-\frac{1}{2(\ln x)^2}\right)\Bigg|_N^e$$

$$= \lim_{N\to 1^+} \left(-\frac{1}{2} + \frac{1}{2(\ln N)^2}\right)$$

$= \infty$ DIVERGES

SO $\displaystyle\int_1^\infty \frac{1}{x(\ln x)^3}\,dx$ DIVERGES

Evaluate $\displaystyle\int \frac{\sqrt{x^2-4}}{x^2}\,dx$.

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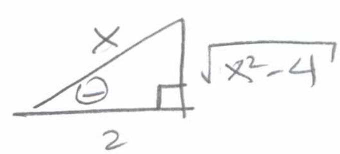
$x = 2\sec\theta \longrightarrow \sec\theta = \frac{x}{2}$

$dx = 2\sec\theta\tan\theta\,d\theta$



$$\int \frac{\cancel{2}\tan\theta}{4\sec^{\cancel{2}}\theta}\cdot \cancel{2}\,\cancel{\sec}\theta\tan\theta\,d\theta$$

$$= \int \frac{\tan^2\theta}{\sec\theta}\,d\theta$$

$$= \int \frac{\sec^2\theta - 1}{\sec\theta}\,d\theta$$

$$= \int (\sec\theta - \cos\theta)\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| - \sin\theta + C$$

$$= \ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| - \frac{\sqrt{x^2-4}}{x} + C$$

$$= \ln\left|x + \sqrt{x^2-4}\right| - \frac{\sqrt{x^2-4}}{x} + C$$

Evaluate $\int_1^{\infty} \frac{\ln x}{x^3}\, dx$.

SCORE: _____ / 25 PTS

$$= \lim_{N\to\infty} \int_1^N \frac{\ln x}{x^3}\, dx$$

$$= \lim_{N\to\infty} \left(-\frac{\ln x}{2x^2} - \frac{1}{4x^2} \right) \Big|_1^N$$

$$= \lim_{N\to\infty} \left(-\frac{\ln N}{2N^2} - \frac{1}{4N^2} - \left(0 - \frac{1}{4}\right) \right)$$

$$= 0 - 0 + \frac{1}{4}$$

$$= \frac{1}{4}$$

$u = \ln x \quad dv = x^{-3}\,dx$

$du = x^{-1}\,dx \quad v = -\frac{1}{2}x^{-2}\,dx$

$$-\frac{1}{2}x^{-2}\ln x + \frac{1}{2}\int x^{-3}\,dx$$

$$= -\frac{1}{2}x^{-2}\ln x - \frac{1}{4}x^{-2}$$

$$\lim_{N\to\infty} \frac{\ln N}{2N^2} \qquad \frac{\infty}{\infty}$$

$$= \lim_{N\to\infty} \frac{\frac{1}{N}}{4N} \qquad \frac{0}{\infty}$$

$$= 0$$

Evaluate $\int \frac{36 - x^2}{x^3 - 6x^2 + 9x}\, dx$.

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$$= \int \left(\frac{4}{x} - \frac{5}{x-3} + \frac{9}{(x-3)^2} \right) dx$$

$$= 4\ln|x| - 5\ln|x-3| - \frac{9}{x-3} + C$$

$$\frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} = \frac{36-x^2}{x^3-6x^2+9x}$$

$$A(x-3)^2 + Bx(x-3) + Cx = 36 - x^2$$

$x=0 \quad 9A = 36 \to A = 4$

$x=3 \quad 3C = 27 \to C = 9$

COEF OF x^2 $\quad A + B = -1 \to 4 + B = -1$

$B = -5$

SANITY CHECK

$$x=2 \quad \frac{36-4}{8-24+18} = \frac{32}{2} = 16$$

$$\frac{4}{2} - \frac{5}{-1} + \frac{9}{1} = 2 + 5 + 9$$

$$= 16 \checkmark$$

Determine if $\int_1^{\infty} \frac{3+\cos x}{\sqrt{x}}\,dx$ converges or diverges. **SCORE: _____ / 25 PTS**

$-1 \le \cos x \le 1$

$2 \le 3+\cos x \le 4$

$0 \le \frac{2}{\sqrt{x}} \le \frac{3+\cos x}{\sqrt{x}} \le \frac{4}{\sqrt{x}} \longrightarrow 0 \le \frac{1}{\sqrt{x}} \le \frac{3+\cos x}{\sqrt{x}}$

$\int_1^{\infty} \frac{1}{\sqrt{x}}\,dx$ DIVERGES SINCE $p = \frac{1}{2} \le 1$

SO $\int_1^{\infty} \frac{3+\cos x}{\sqrt{x}}\,dx$ DIVERGES

Evaluate $\int \arctan \sqrt[3]{x}\,dx$. **SCORE: _____ / 25 PTS**

$u = \sqrt[3]{x}$

$x = u^3$

$dx = 3u^2\,du$

$\int 3u^2 \arctan u\,du$

u	dV
$\arctan u$	$3u^2$
$\frac{1}{1+u^2}$	u^3

$$\begin{array}{r} u \\ u^2+1 \overline{)\, u^3 \qquad} \\ \underline{u^3+u} \\ -u \end{array}$$

$= u^3 \arctan u - \int \frac{u^3}{1+u^2}\,du$

$= u^3 \arctan u - \int \left(u - \frac{u}{1+u^2}\right) du$

$v = 1+u^2$

$dv = 2u\,du$

$\int \frac{1}{2} \frac{1}{v}\,dv = \frac{1}{2}\ln|v|$

$= \frac{1}{2}\ln(1+u^2)$

$= u^3 \arctan u - \left(\frac{1}{2}u^2 - \frac{1}{2}\ln(1+u^2)\right) + C$

$= x \arctan x^{\frac{1}{3}} - \frac{1}{2}x^{\frac{2}{3}} - \frac{1}{2}\ln(1+x^{\frac{2}{3}}) + C$