Math 1B Midterm 3 Review

For sections 5.4–5.5 and 7.1–7.5:

Get together a group of classmates.

Make a copy of the following pages and integrals:

5.4	5-46	7.1	3-45	7.4	7-54
5.5	7-73	7.2	1-35, 47-49, 51-52	7.5	1-82
5.REV	9-40	7.3	4-29	7.REV	1-38

Cut them up, so each question is on a separate slip of paper.

Throw them in a pile and mix them up.

One at a time, randomly pick out a question from the pile and everyone solve it independently.

Compare solutions and discuss which one is fastest/easiest, and how you can recognize what method to use.

The following questions act as a review for 7.8.

[1] Determine if the following integrals converge or diverge. If an integral converges, find its value.

$$[a] \qquad \int_{0}^{\infty} x^{2} e^{-3x} dx \qquad [b] \qquad \int_{0}^{\infty} \frac{1}{\sqrt[3]{x-1}} dx \qquad [c] \qquad \int_{-\infty}^{\infty} \frac{1}{x^{2}+4} dx \qquad [d] \qquad \int_{-\infty}^{\infty} \frac{x}{x^{2}+4} dx$$

$$[e] \qquad \int_{-\infty}^{0} \frac{e^{x}}{1+e^{x}} dx \qquad [f] \qquad \int_{0}^{2} \frac{1}{\sqrt{4-x^{2}}} dx \qquad [g] \qquad \int_{0}^{2} \frac{x}{\sqrt{4-x^{2}}} dx \qquad [h] \qquad \int_{0}^{1} \frac{1}{x(\ln x)^{2}} dx$$

$$[i] \qquad \int_{0}^{\pi} \tan x \, dx \qquad$$

[2] Determine if the following integrals converge or diverge. Justify your answer.

[a]
$$\int_{1}^{\infty} \frac{2 + \sin x}{x} dx$$
 [b] $\int_{1}^{\infty} \frac{2 + \sin x}{x^2} dx$ [c] $\int_{0}^{\infty} e^{-x^2 dx}$ [d] $\int_{e}^{\infty} \frac{1}{\ln x} dx$
[e] $\int_{e}^{\infty} \frac{1}{x \ln x} dx$ [f] $\int_{2}^{\infty} \frac{x + 1}{\sqrt{x^4 - 1}} dx$ [g] $\int_{1}^{\infty} \frac{\cos^2 x}{x e^x} dx$

Answers

- $\frac{2}{27}$ $\frac{\pi}{2}$ [1] [b] diverges [c] [d] diverges [a] $\frac{\pi}{2}$ 2 $\ln 2$ diverges [e] [f] [g] [h]
 - [i] diverges

[2] [a] diverges – compare to
$$\frac{1}{x}$$
 [b] converges – compare to $\frac{3}{x^2}$
[c] converges – compare to e^{-x} [d] diverges – compare to $\frac{1}{x}$
[e] diverges – find antiderivative and take limit [f] diverges – compare to $\frac{1}{x}$

[g] converges – compare to $\frac{1}{e^x}$