SCORE: _____ / 20 POINTS + _____ / 10 POINTS FROM GREENSHEET QUIZ

NO CALCULATORS ALLOWED
SHOW PROPER WORK & SIMPLIFY ALL ANSWERS
(ANSWERS WITHOUT SOLUTIONS WILL *NOT* EARN FULL CREDIT)

Find $\frac{d}{dx}\sinh^{-1}(\operatorname{sech} x)$. $\int \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$

$1-\tanh^2 x = \operatorname{sech}^2 x$
$-\tanh^2 x = \operatorname{sech}^2 x - 1$
$\tanh^2 x = 1 - \operatorname{sech}^2 x$

SCORE: $2\frac{1}{2}$ / 3 PTS

$= \frac{1}{\sqrt{1+(\operatorname{sech}^2 x)}} \cdot (-\operatorname{sech} x \tanh x)$ (1½) (1)

$= \frac{1}{\tanh x} \cdot -\operatorname{sech} x \tanh x$

$= \boxed{-\operatorname{sech} x}$

Prove the derivative of $\coth^{-1} x$. **Do NOT use any other inverse hyperbolic functions in your proof.** SCORE: $4\frac{1}{2}$ / 5 PTS
You may use any of the other identities or derivatives of (non-inverse) hyperbolic functions that were listed in your textbook without proving them. **NOTE: The Pythagorean–like identity for coth x must be proven if you wish to use it.**

$\int \coth^{-1} x = \frac{1}{1-x^2}$

$\coth^2 y - 1 = \operatorname{csch}^2 y$ (½)

$y = \coth^{-1} x$
$x = \coth y$ (1)

$-\operatorname{csch}^2 y \, y' = 1$ (1½)

$y' = \frac{1}{-\operatorname{csch}^2 y} = \frac{-1}{\coth^2 y - 1} = \boxed{\frac{-1}{x^2-1}} = \boxed{\frac{1}{1-x^2}}$ (½) (1)

Using the definition of "area under a function" given in class, write an algebraic expression for the area under $f(x) = \sqrt{7x+2}$ over the interval $[1, 4]$. **Do NOT evaluate the expression. You do NOT need to draw a graph to explain your answer.** SCORE: 3 / 3 PTS

$f(x) = \sqrt{7x+2}$ $[1, 4]$ $f(1+i\Delta x)\Delta x$ $\Delta x = \frac{b-a}{n} = \frac{3}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{7\left(1+\frac{3i}{n}\right)+2}\right)\left(\frac{3}{n}\right)$ (½) (½) (1) (1)

Find $\lim_{x \to 0^-} \operatorname{csch} x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.**

SCORE: _____ / 2 PTS

$\lim_{x \to 0^-} \operatorname{csch} x$ (*)

$\frac{1}{\sinh(0)} = \frac{1}{0} =$ und.

$\frac{1}{\sinh(0^-)} = \frac{1}{-\#} = -\infty$ (1)

Prove the logarithmic formula for $\sinh^{-1} x$.

SCORE: 0 / 4 PTS

$y = \sinh^{-1} x$

$x = \sinh y$

$\cosh y \, y' = 1$

$y' = \frac{1}{\cosh y} = \frac{1}{\sqrt{1 + \sinh^2 y}} = \frac{1}{\sqrt{1 + x^2}}$

$\cosh^2 y - \sinh^2 y = 1$

$\cosh y = \sqrt{1 + \sinh^2 y}$

Estimate the area under the function shown on the right over the interval $[-3, 5]$ using the right hand sum with 4 equal width subintervals.

SCORE: _____ / 3 PTS

(*)

$2 \cdot f(-1) = 2$

$2 \cdot f(1) = 2$

$2 \cdot f(3) = 8$

$2 \cdot f(5) = 2$

14 estimated area between $[-3, 5]$

$\frac{1}{2}$ $\frac{1}{2}$