Find $\frac{d}{dx}\sinh^{-1}(\operatorname{sech} x)$.

SCORE: _____ / 3 PTS

$= \frac{1}{\sqrt{1+\operatorname{sech}^2 x}} \cdot -\operatorname{sech} x \tanh x$ (1) (1½)

$= -\frac{\operatorname{sech} x \tanh x}{\sqrt{1+\operatorname{sech}^2 x}}$ (½)

GIVE YOURSELF FULL CREDIT IF YOU SKIPPED THIS LINE, BUT GOT CORRECT FINAL ANSWER

Prove the derivative of $\coth^{-1} x$. **Do NOT use any other inverse hyperbolic functions in your proof.** **SCORE: _____ / 5 PTS**

You may use any of the other identities or derivatives of (non-inverse) hyperbolic functions that were listed in your textbook without proving them. **NOTE: The Pythagorean–like identity for $\coth x$ must be proven if you wish to use it.**

$y = \coth^{-1} x$

$\coth y = x$ (1)

$\frac{d}{dx} \coth y = \frac{d}{dx} x$

$-\operatorname{csch}^2 y \frac{dy}{dx} = 1$ (1 1/2)

$\frac{dy}{dx} = \frac{1}{-\operatorname{csch}^2 y} = \frac{1}{1 - x^2}$

(1/2) (1)

$\cosh^2 y - \sinh^2 y = 1$

(1/2) $\frac{\cosh^2 y}{\sinh^2 y} - \frac{\sinh^2 y}{\sinh^2 y} = \frac{1}{\sinh^2 y}$

(1/2) $\coth^2 y - 1 = \operatorname{csch}^2 y$

$x^2 - 1 = \operatorname{csch}^2 y$

Using the definition of "area under a function" given in class, write an algebraic expression for the area under $f(x) = \sqrt{7x+2}$ over the interval $[1, 4]$. **Do NOT evaluate the expression. You do NOT need to draw a graph to explain your answer.**

SCORE: _____ / 3 PTS

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(1 + i\Delta x)\Delta x \qquad \Delta x = \frac{4-1}{n} = \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + \frac{3i}{n}\right)\frac{3}{n}$$

$$= \left[\lim_{n\to\infty}\right]\left[\sum_{i=1}^{n}\right]\left[\frac{3}{n}\right]\left[\sqrt{7\left(1+\frac{3i}{n}\right)+2}\right] = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{9 + \frac{21i}{n}}$$

(1/2) (1/2) (1) (1)

SUBTRACT (1/2) POINT IF YOU TRIED TO SIMPLIFY BUT DID NOT GET THIS

Find $\lim_{x \to 0} \operatorname{csch} x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.** **SCORE: _____ / 2 PTS**

$$\lim_{x \to 0^-} \frac{2}{e^x - e^{-x}} = -\infty \quad (1)$$

IF $x < 0$, $e^x < e^{-x}$

$$e^x - e^{-x} < 0$$

AS $x \to 0$, $e^x - e^{-x} \to 0^-$ (1), $\frac{2}{e^x - e^{-x}} \to \frac{2}{0^-} \to -\infty$

Prove the logarithmic formula for $\sinh^{-1} x$.

SCORE: _____ / 4 PTS

$y = \sinh^{-1} x$

$\sinh y = x$

$x = \frac{e^y - e^{-y}}{2}$ (1/2)

LET $z = e^y$

$x = \frac{z - \frac{1}{z}}{2}$ (1/2)

$2x = z - \frac{1}{z}$

$2xz = z^2 - 1$ (1)

$0 = z^2 - 2xz - 1$

$z = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$ (1/2)

$z = \frac{2x \pm 2\sqrt{x^2+1}}{2}$

$z = x \pm \sqrt{x^2+1} = e^y$ (1/2)

$\sqrt{x^2+1} > x \rightarrow x - \sqrt{x^2+1} < 0$ (1/2)

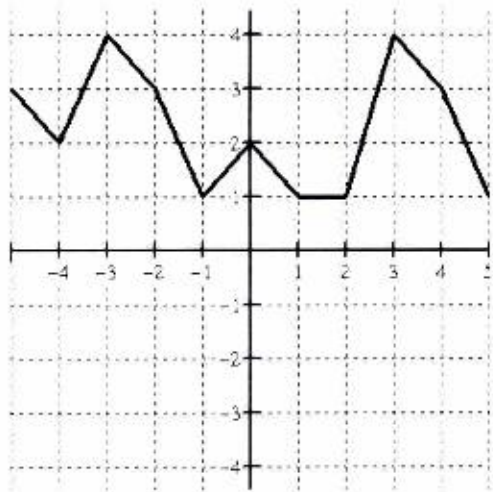
BUT $e^y > 0$

SO $e^y = x + \sqrt{x^2+1}$

$y = \ln(x + \sqrt{x^2+1}) = \sinh^{-1} x$ (1/2)

Estimate the area under the function shown on the right over the interval $[-3, 5]$ using the right hand sum with 4 equal width subintervals.

SCORE: _____ / 3 PTS



$\Delta x = \frac{5-(-3)}{4} = 2$

RIGHT ENDPOINTS $x = -1, 1, 3, 5$

$f(-1)\Delta x + f(1)\Delta x + f(3)\Delta x + f(5)\Delta x$

$= (1)(2) + (1)(2) + (4)(2) + (1)(2)$

$= (1+1+4+1)(2)$

$= 14$

$\left(\frac{1}{2}\right)$ POINT EACH