Find $\lim_{x \to 0^-} \operatorname{csch} x$. **<u>Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.</u>** SCORE: ______ / 2 PTS

$$\lim_{x \to 0^-} \frac{2}{e^x - e^{-x}} = \underline{-\infty} \quad 1$$

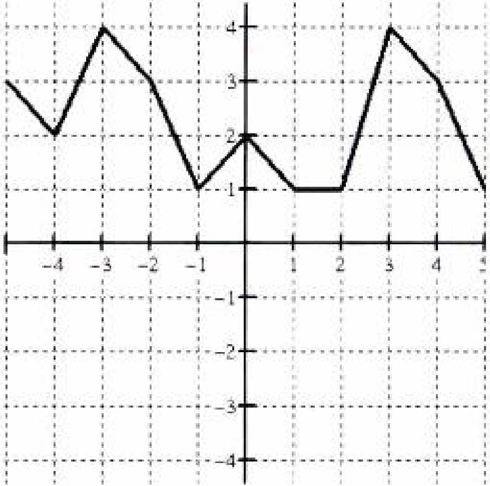
IF $x < 0$, $e^x < e^{-x}$

$e^x - e^{-x} < 0$

AS $x \to 0$, $\underline{e^x - e^{-x} \to 0^-}$ 1, $\frac{2}{e^x - e^{-x}} \to \frac{2}{0^-} \to -\infty$

Estimate the area under the function shown on the right over the interval $[-5, 3]$ using the left hand sum with 4 equal width subintervals.

SCORE: ______ / 3 PTS



$\Delta x = \frac{3 - -5}{4} = 2$

LEFT ENDPOINTS $x = -5, -3, -1, 1$

$f(-5)\Delta x + f(-3)\Delta x + f(-1)\Delta x + f(1)\Delta x$

$= (3)(2) + (4)(2) + (1)(2) + (1)(2)$

$= (\underline{3} + \underline{4} + \underline{1} + \underline{1})(\underline{2})$ ½ POINT EACH

$= \underline{18}$

Prove the logarithmic formula for $\sinh^{-1} x$. **SCORE: ______ / 4 PTS**

$y = \sinh^{-1} x$

$\sinh y = x$

$x = \frac{e^y - e^{-y}}{2}$ $\frac{1}{2}$

LET $z = e^y$

$x = \frac{z - \frac{1}{z}}{2}$ $\frac{1}{2}$

$2x = z - \frac{1}{z}$

$2xz = z^2 - 1$ 1

$0 = z^2 - 2xz - 1$

$\frac{1}{2}$ $z = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$

$z = \frac{2x \pm 2\sqrt{x^2+1}}{2}$

$\frac{1}{2}$ $z = x \pm \sqrt{x^2+1} = e^y$

$\sqrt{x^2+1} > x \longrightarrow x - \sqrt{x^2+1} < 0$ $\frac{1}{2}$

BUT $e^y > 0$

SO $e^y = x + \sqrt{x^2+1}$

$y = \frac{1}{2}\ \ln\left(x + \sqrt{x^2+1}\right) = \sinh^{-1} x$

Find $\frac{d}{dx}\sinh^{-1}(\operatorname{sech} x)$.

SCORE: ______ / 3 PTS

$$= \boxed{\frac{1}{\sqrt{1+\operatorname{sech}^2 x}}} \cdot \underline{-\operatorname{sech} x \tanh x}$$

$1\frac{1}{2}$ 1

$$= \underline{-\frac{\operatorname{sech} x \tanh x}{\sqrt{1+\operatorname{sech}^2 x}}}$$

$\frac{1}{2}$

GIVE YOURSELF FULL CREDIT IF YOU SKIPPED THIS LINE, BUT GOT CORRECT FINAL ANSWER

Prove the derivative of $\coth^{-1} x$. **Do NOT use any other inverse hyperbolic functions in your proof**. SCORE: ______ / 5 PTS

You may use any of the other identities or derivatives of (non-inverse) hyperbolic functions that were listed in your textbook without proving them. **NOTE: The Pythagorean–like identity for $\coth x$ must be proven if you wish to use it.**

$$y = \coth^{-1} x$$

$$\underbrace{\coth y = x}_{}, \quad 1$$

$$\frac{d}{dx} \coth y = \frac{d}{dx} x$$

$$\underbrace{-\operatorname{csch}^2 y \frac{dy}{dx} = 1}_{}, \quad 1\tfrac{1}{2}$$

$$\underbrace{\frac{dy}{dx} = \frac{1}{-\operatorname{csch}^2 y}}_{\frac{1}{2}} = \underbrace{\frac{1}{1 - x^2}}_{1}$$

$$\cosh^2 y - \sinh^2 y = 1$$

$$\tfrac{1}{2} \quad \underbrace{\frac{\cosh^2 y}{\sinh^2 y} - \frac{\sinh^2 y}{\sinh^2 y} = \frac{1}{\sinh^2 y}}_{},$$

$$\tfrac{1}{2} \quad \underbrace{\coth^2 y - 1 = \operatorname{csch}^2 y}_{},$$

$$x^2 - 1 = \operatorname{csch}^2 y$$

Using the definition of "area under a function" given in class, write an algebraic expression for the area under **SCORE: ______ / 3 PTS**
$f(x) = \sqrt{3x+1}$ over the interval $[2, 7]$. **<u>Do NOT evaluate the expression. You do NOT need to draw a graph to explain your answer.</u>**

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(2+i\Delta x)\Delta x \qquad \Delta x = \frac{7-2}{n} = \frac{5}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(2+\frac{5i}{n}\right)\frac{5}{n}$$

$$= \underbrace{\boxed{\lim_{n\to\infty}}}_{\frac{1}{2}} \underbrace{\boxed{\sum_{i=1}^{n}}}_{\frac{1}{2}} \underbrace{\boxed{\frac{5}{n}}}_{1} \underbrace{\sqrt{3\left(2+\frac{5i}{n}\right)+1}}_{1} \quad \text{OR} \quad \lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n}\sqrt{7+\frac{15i}{n}}$$

SUBTRACT $\frac{1}{2}$ POINT IF YOU TRIED TO SIMPLIFY BUT DID <u>NOT</u> GET THIS