Using the definition of "area under a function" given in class, write an algebraic expression for the area under $f(x) = \sqrt{2x+3}$ over the interval $[1, 7]$. **<u>Do NOT evaluate the expression. You do NOT need to draw a graph to explain your answer.</u>** **SCORE: ______ / 3 PTS**

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(1 + i\Delta x)\Delta x \qquad \Delta x = \frac{7-1}{n} = \frac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + \frac{6i}{n}\right)\frac{6}{n}$$

$$= \underbrace{\boxed{\lim_{n\to\infty}}}_{\frac{1}{2}} \underbrace{\boxed{\sum_{i=1}^{n}}}_{\frac{1}{2}} \underbrace{\boxed{\frac{6}{n}}}_{1} \sqrt{\underbrace{2\left(1+\frac{6i}{n}\right)+3}_{1}} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n}\sqrt{5 + \frac{12i}{n}}$$

SUBTRACT $\frac{1}{2}$ POINT IF YOU TRIED TO SIMPLIFY BUT DID NOT GET THIS

Find $\frac{d}{dx}\sinh^{-1}(\operatorname{csch} x)$ ... SCORE: ... / 3 PTS

Find $\frac{d}{dx}\sinh^{-1}(\operatorname{sech} x)$.

SCORE: ______ / 3 PTS

$$= \boxed{\frac{1}{\sqrt{1+\operatorname{sech}^2 x}}} \cdot \underline{-\operatorname{sech} x \tanh x} \quad 1$$

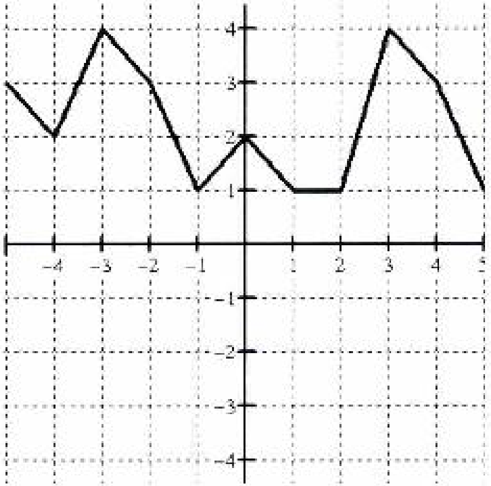
$1\frac{1}{2}$

$$= \underline{-\frac{\operatorname{sech} x \tanh x}{\sqrt{1+\operatorname{sech}^2 x}}} \quad \frac{1}{2}$$

GIVE YOURSELF FULL CREDIT IF YOU SKIPPED THIS LINE, BUT GOT CORRECT FINAL ANSWER

Estimate the area under the function shown on the right over the interval $[-5, 3]$ using the right hand sum with 4 equal width subintervals.

SCORE: ______ / 3 PTS



$\Delta x = \frac{3 - -5}{4} = 2$

RIGHT ENDPOINTS $x = -3, -1, 1, 3$

$f(-3)\Delta x + f(-1)\Delta x + f(1)\Delta x + f(3)\Delta x$

$= (4)(2) + (1)(2) + (1)(2) + (4)(2)$

$= (\underline{4} + \underline{1} + \underline{1} + \underline{4})(\underline{2})$

$= \underline{20}$

($\frac{1}{2}$) POINT EACH

Find $\lim_{x \to 0^-} \operatorname{csch} x$. **<u>Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.</u>** SCORE: ______ / 2 PTS

$$\lim_{x \to 0^-} \frac{2}{e^x - e^{-x}} = -\infty \quad 1$$

IF $x < 0$, $e^x < e^{-x}$

$e^x - e^{-x} < 0$

AS $x \to 0$, $e^x - e^{-x} \to 0^-$ 1, $\frac{2}{e^x - e^{-x}} \to \frac{2}{0^-} \to -\infty$

Prove the logarithmic formula for $\sinh^{-1} x$. **SCORE: ______ / 4 PTS**

$y = \sinh^{-1} x$

$\sinh y = x$

$x = \dfrac{e^y - e^{-y}}{2}$ $\frac{1}{2}$

LET $z = e^y$

$x = \dfrac{z - \frac{1}{z}}{2}$ $\frac{1}{2}$

$2x = z - \frac{1}{z}$

$2xz = z^2 - 1$ 1

$0 = z^2 - 2xz - 1$

$\frac{1}{2}$ $z = \dfrac{2x \pm \sqrt{4x^2 + 4}}{2}$

$z = \dfrac{2x \pm 2\sqrt{x^2 + 1}}{2}$

$\frac{1}{2}$ $z = x \pm \sqrt{x^2 + 1} = e^y$

$\sqrt{x^2+1} > x \longrightarrow x - \sqrt{x^2+1} < 0$ $\frac{1}{2}$

BUT $e^y > 0$

SO $e^y = x + \sqrt{x^2 + 1}$

$y =$ $\frac{1}{2}$ $\ln\left(x + \sqrt{x^2+1}\right) = \sinh^{-1} x$

Prove the derivative of $\coth^{-1} x$. **Do NOT use any other inverse hyperbolic functions in your proof.** **SCORE: ______ / 5 PTS**
You may use any of the other identities or derivatives of (non-inverse) hyperbolic functions that were listed in your textbook without proving them. **NOTE: The Pythagorean–like identity for $\coth x$ must be proven if you wish to use it.**

$$y = \coth^{-1} x$$

$$\coth y = x \quad 1$$

$$\frac{d}{dx} \coth y = \frac{d}{dx} x$$

$$-\operatorname{csch}^2 y \frac{dy}{dx} = 1 \quad 1\tfrac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{-\operatorname{csch}^2 y} = \frac{1}{1-x^2}$$

$\tfrac{1}{2}$ $\quad$ 1

$$\cosh^2 y - \sinh^2 y = 1$$

$$\tfrac{1}{2} \quad \frac{\cosh^2 y}{\sinh^2 y} - \frac{\sinh^2 y}{\sinh^2 y} = \frac{1}{\sinh^2 y}$$

$$\tfrac{1}{2} \quad \coth^2 y - 1 = \operatorname{csch}^2 y$$

$$x^2 - 1 = \operatorname{csch}^2 y$$