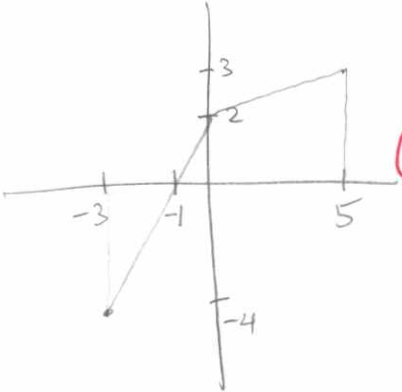
Use geometry to find $\int_{-3}^{5} f(x)\,dx$ if $f(x) = \begin{cases} 2x+2, & x<0 \\ \frac{x}{5}+2, & x \geq 0 \end{cases}$.

SCORE: ______ / 5 PTS

<u>For full credit, you must clearly show the use of geometry formulae, not just the final answer.</u>
<u>NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.</u>



$-\frac{1}{2}(4)(-1-{-3}) + \frac{1}{2}(2)(0-{-1}) + \frac{1}{2}(2+3)(5-0)$

DON'T FORGET THE NEGATIVE

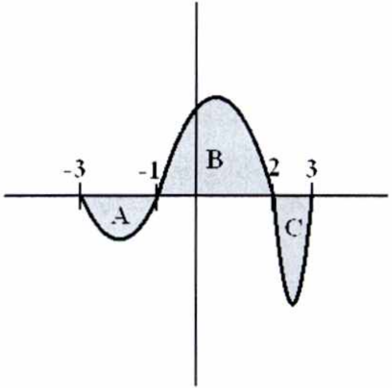
$= \underbrace{-\frac{1}{2}(4)(2)}_{1\frac{1}{2}} + \underbrace{\frac{1}{2}(2)(1)}_{1} + \underbrace{\frac{1}{2}(5)(5)}_{1\frac{1}{2}}$

$= -4 + 1 + \frac{25}{2}$

$= \underbrace{\frac{19}{2}}_{1}$

The graph of $f(x)$ is shown on the right. If the area of shaded region A is 6, the area of shaded region B is 18, and the area of shaded region C is 5, find $\int_{-3}^{3} (4 - 2f(x))\, dx$.

SCORE: ______ / 5 PTS



<u>**For full credit, you must clearly show the use of all necessary properties of the definite integral.**</u>

$= \int_{-3}^{3} 4\,dx - 2\int_{-3}^{3} f(x)\,dx$ $1\frac{1}{2}$

$= 4(3 - -3) - 2\left[\int_{-3}^{-1} f(x)\,dx + \int_{-1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx\right]$

1 OR

$= 4(6) - 2[-6 + 18 - 5]$ $1\frac{1}{2}$

$= 24 - 2(7)$

$= 10$ 1

Use the definition of the definite integral and right hand sums to evaluate $\int_3^5 (x^2 - 8x + 15)\, dx$. **SCORE: ______ / 7 PTS**

<u>NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.</u>

$$\Delta x = \frac{5-3}{n} = \frac{2}{n}$$

$$\frac{1}{2} \quad \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(3+\frac{2i}{n}\right)^2 - 8\left(3+\frac{2i}{n}\right) + 15 \right) \frac{2}{n} \quad \frac{1}{2}$$

1

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(9 + \frac{12i}{n} + \frac{4i^2}{n^2} - 24 - \frac{16i}{n} + 15 \right) \frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} - \frac{4i}{n} \right) \quad 1$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4}{n^2} \sum_{i=1}^{n} i^2 - \frac{4}{n} \sum_{i=1}^{n} i \right) \quad 1$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4}{n^2} \frac{n(n+1)(2n+1)}{6} - \frac{4}{n} \frac{n(n+1)}{2} \right)$$

1 1

$$= \lim_{n\to\infty} \left(\frac{4(n+1)(2n+1)}{3n^2} - \frac{4(n+1)}{n} \right)$$

$$= \frac{8}{3} - 4 = -\frac{4}{3} \quad 1$$

Sketch a region whose area is given by the expression $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{7}{n}\sqrt{9+\frac{7i}{n}}$. **SCORE: ______ / 4 PTS**

$\int_9^{16}\sqrt{x}\,dx$

1 FOR GRAPH + SHADING TOGETHER

$f(x)=\sqrt{x}$

1

$\Delta x=\frac{7}{n}=\frac{b-a}{n} \rightarrow b-a=7$

$9+\frac{7i}{n}=a+i\Delta x \rightarrow a=9$

$b=16$

Find $\int_1^4 \frac{2x^2 - \sqrt{x}}{x^3}\, dx$. **SCORE: ______ / 5 PTS**

$= \int_1^4 \left(\frac{2}{x} - x^{-\frac{5}{2}}\right) dx$ 1

$= \left(2\ln|x| + \frac{2}{3}x^{-\frac{3}{2}}\right)\Big|_1^4$

1 $\frac{1}{2}$

$= 2\ln|4| + \frac{2}{3}\left(\frac{1}{8}\right) - \left(2\ln|1| + \frac{2}{3}(1)\right)$

$= 2\ln 4 + \frac{1}{12} - \frac{2}{3}$ 1

$= 2\ln 4 - \frac{7}{12}$ $\frac{1}{2}$