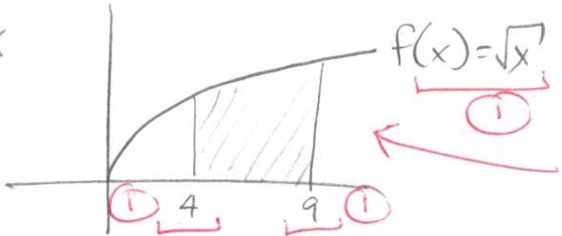
Sketch a region whose area is given by the expression $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{5}{n}\sqrt{4+\frac{5i}{n}}$.

SCORE: ______ / 4 PTS

$\int_4^9 \sqrt{x}\,dx$



$\Delta x = \frac{5}{n} = \frac{b-a}{n} \to b-a=5$

$4+\frac{5i}{n} = a+i\Delta x \to a=4$

$b=9$

1 FOR GRAPH + SHADING TOGETHER

Use the definition of the definite integral and right hand sums to evaluate $\int_{-1}^{2} (x^2 - x - 2)\, dx$. **SCORE: ______ / 7 PTS**

NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.

$$\Delta x = \frac{2 - -1}{n} = \frac{3}{n}$$

$$\frac{1}{2} \quad \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n}} \underbrace{\left(\left(-1+\frac{3i}{n}\right)^2 - \left(-1+\frac{3i}{n}\right) - 2\right)}_{1} \underbrace{\frac{3}{n}} \quad \frac{1}{2}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{6i}{n} + \frac{9i^2}{n^2} + 1 - \frac{3i}{n} - 2\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \underbrace{\left(\frac{9i^2}{n^2} - \frac{9i}{n}\right)} \quad 1$$

$$= \lim_{n\to\infty} \frac{3}{n} \underbrace{\left(\frac{9}{n^2}\sum_{i=1}^{n} i^2 - \frac{9}{n}\sum_{i=1}^{n} i\right)} \quad 1$$

$$= \lim_{n\to\infty} \frac{3}{n} \left(\underbrace{\frac{9}{n^2}\frac{n(n+1)(2n+1)}{6}}_{1} - \underbrace{\frac{9}{n}\frac{n(n+1)}{2}}_{1}\right)$$

$$= \lim_{n\to\infty} \left(\frac{9(n+1)(2n+1)}{2n^2} - \frac{27(n+1)}{2n}\right)$$

$$= 9 - \frac{27}{2} = \underbrace{-\frac{9}{2}} \quad 1$$

Find $\int_1^4 \frac{2x^2+\sqrt{x}}{x^3}\,dx$.

SCORE: ______ / 5 PTS

$= \int_1^4 \left(\frac{2}{x} + x^{-\frac{5}{2}}\right) dx$ (1)

$= \left(2\ln|x| - \frac{2}{3}x^{-\frac{3}{2}}\right)\Big|_1^4$ (1) $(1\frac{1}{2})$

$= 2\ln|4| - \frac{2}{3}\left(\frac{1}{8}\right) - \left(2\ln|1| - \frac{2}{3}(1)\right)$

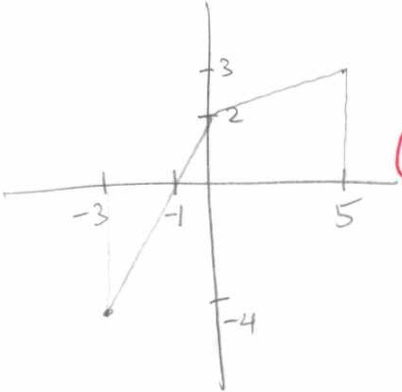
$= 2\ln 4 - \frac{1}{12} + \frac{2}{3}$ (1)

$= 2\ln 4 + \frac{7}{12}$ $(\frac{1}{2})$

Use geometry to find $\int_{-3}^{5} f(x)\,dx$ if $f(x)=\begin{cases}2x+2, & x<0\\ \dfrac{x}{5}+2, & x\geq 0\end{cases}$.

SCORE: ______ / 5 PTS

<u>For full credit, you must clearly show the use of geometry formulae, not just the final answer.</u>
<u>NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.</u>



$-\frac{1}{2}(4)(-1-{}^{-}3)+\frac{1}{2}(2)(0-{}^{-}1)+\frac{1}{2}(2+3)(5-0)$

DON'T FORGET THE NEGATIVE

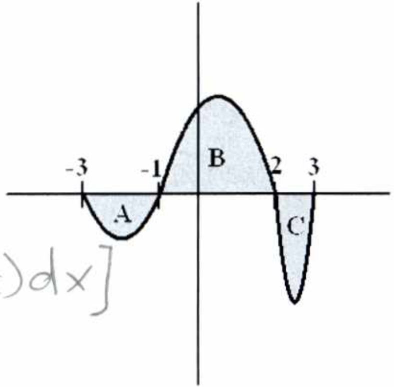
$= \underbrace{-\frac{1}{2}(4)(2)}_{1\frac{1}{2}} + \underbrace{\frac{1}{2}(2)(1)}_{1} + \underbrace{\frac{1}{2}(5)(5)}_{1\frac{1}{2}}$

$= -4+1+\frac{25}{2}$

$= \underbrace{\frac{19}{2}}_{1}$

The graph of $f(x)$ is shown on the right. If the area of shaded region A is 5, the area of shaded region B is 20, and the area of shaded region C is 4, find $\int_{-3}^{3}(7-8f(x))\,dx$.

SCORE: ______ / 5 PTS



For full credit, you must clearly show the use of all necessary properties of the definite integral.

$$= \underbrace{\int_{-3}^{3} 7\,dx - 8\int_{-3}^{3} f(x)\,dx}\ \ 1\tfrac{1}{2}$$

$$= 7(3 - {-3}) - 8\left[\int_{-3}^{-1} f(x)\,dx + \int_{-1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx\right]$$

1 OR

$$= 7(6) - 8\,[\underbrace{-5+20-4}]$$

$1\tfrac{1}{2}$

$$= 42 - 8(11)$$

$$= \underline{-46}$$

1