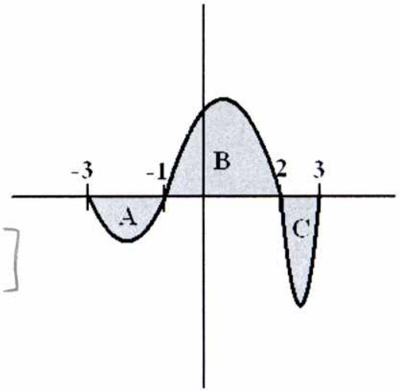
The graph of $f(x)$ is shown on the right. If the area of shaded region A is 6, the area of shaded region B is 18, and the area of shaded region C is 5, find $\int_{-3}^{3}(7-8f(x))\,dx$.

SCORE: ______ / 5 PTS



For full credit, you must clearly show the use of all necessary properties of the definite integral.

$= \underbrace{\int_{-3}^{3} 7\,dx - 8\int_{-3}^{3} f(x)\,dx}$ $1\frac{1}{2}$

1 $= 7(3--3) - 8\left[\int_{-3}^{-1} f(x)\,dx + \int_{-1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx\right]$

OR

$= 7(6) - 8[\underbrace{-6+18-5}]$ $1\frac{1}{2}$

$= 42 - 8(7)$

$= \underbrace{-14}$ 1

Find $\int_1^4 \frac{2x^2 - \sqrt{x}}{x^3}\, dx$.

SCORE: ______ / 5 PTS

$= \int_1^4 \left(\frac{2}{x} - x^{-\frac{5}{2}}\right) dx$ 1

$= \left(2\ln|x| + \frac{2}{3}x^{-\frac{3}{2}}\right)\Big|_1^4$ 1 $\frac{1}{2}$

$= 2\ln|4| + \frac{2}{3}\left(\frac{1}{8}\right) - \left(2\ln|1| + \frac{2}{3}(1)\right)$

$= 2\ln 4 + \frac{1}{12} - \frac{2}{3}$ 1

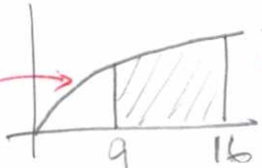
$= 2\ln 4 - \frac{7}{12}$ $\frac{1}{2}$

Sketch a region whose area is given by the expression $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{7}{n}\sqrt{9+\frac{7i}{n}}$.

SCORE: ______ / 4 PTS

$\int_{9}^{16}\sqrt{x}\,dx$

① FOR GRAPH + SHADING TOGETHER



① ①

$\underbrace{f(x)=\sqrt{x}}_{①}$

$\Delta x = \frac{7}{n} = \frac{b-a}{n} \rightarrow b-a=7$

$9+\frac{7i}{n} = a+i\Delta x \rightarrow a=9$

$b=16$

Use the definition of the definite integral and right hand sums to evaluate $\int_3^5 (x^2 - 8x + 15)\, dx$. **SCORE: ______ / 7 PTS**

NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.

$$\Delta x = \frac{5-3}{n} = \frac{2}{n}$$

$$\frac{1}{2} \quad \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(3+\frac{2i}{n}\right)^2 - 8\left(3+\frac{2i}{n}\right) + 15 \right) \frac{2}{n} \quad \frac{1}{2}$$

$$1$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(9 + \frac{12i}{n} + \frac{4i^2}{n^2} - 24 - \frac{16i}{n} + 15 \right) \frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} - \frac{4i}{n} \right) \quad 1$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4}{n^2} \sum_{i=1}^{n} i^2 - \frac{4}{n} \sum_{i=1}^{n} i \right) \quad 1$$

$$= \lim_{n\to\infty} \frac{2}{n} \left(\frac{4}{n^2} \frac{n(n+1)(2n+1)}{6} - \frac{4}{n} \frac{n(n+1)}{2} \right)$$

$$1 \qquad 1$$

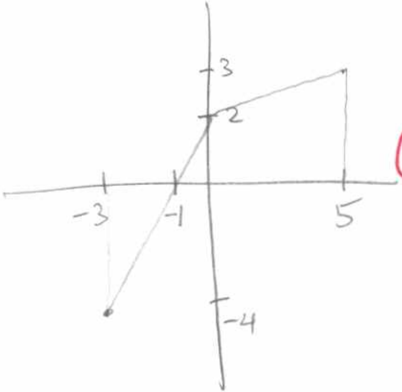
$$= \lim_{n\to\infty} \left(\frac{4(n+1)(2n+1)}{3n^2} - \frac{4(n+1)}{n} \right)$$

$$= \frac{8}{3} - 4 = -\frac{4}{3} \quad 1$$

Use geometry to find $\int_{-3}^{5} f(x)\,dx$ if $f(x) = \begin{cases} 2x+2, & x<0 \\ \dfrac{x}{5}+2, & x \ge 0 \end{cases}$.

SCORE: ______ / 5 PTS

<u>For full credit, you must clearly show the use of geometry formulae, not just the final answer.</u>
<u>NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.</u>



$-\frac{1}{2}(4)(-1-{-3}) + \frac{1}{2}(2)(0-{-1}) + \frac{1}{2}(2+3)(5-0)$

DON'T FORGET THE NEGATIVE

$1\frac{1}{2}$ $= \underbrace{-\frac{1}{2}(4)(2)}_{} + \underbrace{\frac{1}{2}(2)(1)}_{1} + \underbrace{\frac{1}{2}(5)(5)}_{1\frac{1}{2}}$

$= -4 + 1 + \frac{25}{2}$

$= \underbrace{\frac{19}{2}}_{1}$