Use the definition of the definite integral and right hand sums to evaluate $\int_{-1}^{2} (x^2 - x - 2)\, dx$.

SCORE: _____ / 7 PTS

NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.

$$\Delta x = \frac{2-(-1)}{n} = \frac{3}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(-1 + \frac{3i}{n}\right)^2 - \left(-1 + \frac{3i}{n}\right) - 2 \right) \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{6i}{n} + \frac{9i^2}{n^2} + 1 - \frac{3i}{n} - 2 \right) \frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(\frac{9i^2}{n^2} - \frac{9i}{n} \right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \left(\frac{9}{n^2} \sum_{i=1}^{n} i^2 - \frac{9}{n} \sum_{i=1}^{n} i \right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \left(\frac{9}{n^2} \frac{n(n+1)(2n+1)}{6} - \frac{9}{n} \frac{n(n+1)}{2} \right)$$

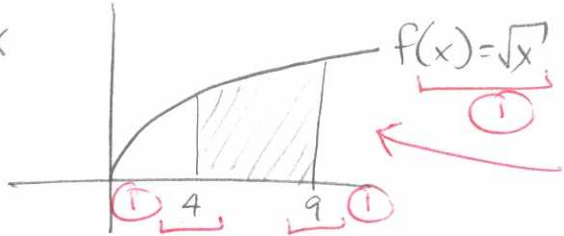
$$= \lim_{n\to\infty} \left(\frac{9(n+1)(2n+1)}{2n^2} - \frac{27(n+1)}{2n} \right)$$

$$= 9 - \frac{27}{2} = -\frac{9}{2}$$

Sketch a region whose area is given by the expression $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{5}{n}\sqrt{4+\frac{5i}{n}}$.

SCORE: _____ / 4 PTS

$\int_4^9 \sqrt{x}\,dx$



$\Delta x = \frac{5}{n} = \frac{b-a}{n} \rightarrow b-a=5$

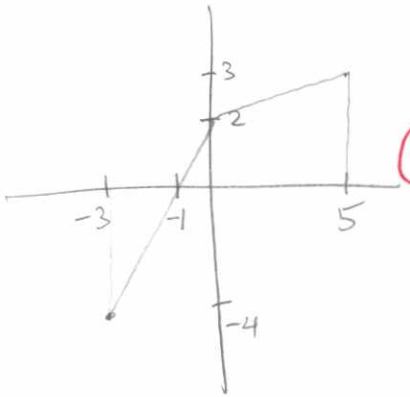
$4+\frac{5i}{n} = a+i\Delta x \rightarrow$ $a=4$, $b=9$

1 FOR GRAPH + SHADING TOGETHER

Use geometry to find $\int_{-3}^{5} f(x)\,dx$ if $f(x) = \begin{cases} 2x+2, & x<0 \\ \frac{x}{5}+2, & x \geq 0 \end{cases}$.

SCORE: _____ / 5 PTS

For full credit, you must clearly show the use of geometry formulae, not just the final answer.
NOTE: You will earn only 1 point if you use the Fundamental Theorem of Calculus instead.



$= -\frac{1}{2}(4)(-1-(-3)) + \frac{1}{2}(2)(0-(-1)) + \frac{1}{2}(2+3)(5-0)$

DON'T FORGET THE NEGATIVE

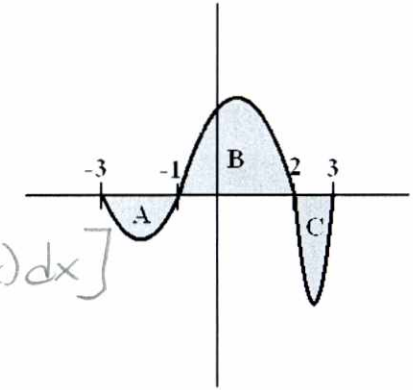
$= \underbrace{-\frac{1}{2}(4)(2)}_{1\frac{1}{2}} + \underbrace{\frac{1}{2}(2)(1)}_{1} + \underbrace{\frac{1}{2}(5)(5)}_{1\frac{1}{2}}$

$= -4 + 1 + \frac{25}{2}$

$= \underbrace{\frac{19}{2}}_{1}$

The graph of $f(x)$ is shown on the right. If the area of shaded region A is 5, the area of shaded region B is 20, and the area of shaded region C is 4, find $\int_{-3}^{3}(4-2f(x))\,dx$.

SCORE: _____ / 5 PTS



For full credit, you must clearly show the use of all necessary properties of the definite integral.

$= \int_{-3}^{3} 4\,dx - 2\int_{-3}^{3} f(x)\,dx$ (1½)

$= [4(3--3) - 2\left[\int_{-3}^{-1} f(x)\,dx + \int_{-1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx\right]$ (1)

OR

$= [4(6) - 2[-5+20-4]$ (1½)

$= 24 - 2(11)$

$= 2$ (1)

Find $\int_1^4 \frac{2x^2+\sqrt{x}}{x^3}\,dx$.

SCORE: _____ / 5 PTS

$= \int_1^4 \left(\frac{2}{x} + x^{-\frac{5}{2}}\right) dx$ (1)

$= \left(2\ln|x| - \frac{2}{3}x^{-\frac{3}{2}}\right)\Big|_1^4$ (1) (1 1/2)

$= 2\ln|4| - \frac{2}{3}\left(\frac{1}{8}\right) - \left(2\ln|1| - \frac{2}{3}(1)\right)$

$= 2\ln 4 - \frac{1}{12} + \frac{2}{3}$ (1)

$= 2\ln 4 + \frac{7}{12}$ (1/2)