Find the error in the logic below. **HINT: It is NOT an arithmetic error.** SCORE: ______ / 2 PTS

$$\int_{-1}^{8} 2x^{-\frac{1}{3}}\,dx \;=\; 3x^{\frac{2}{3}}\Big|_{-1}^{8} \;=\; 3\left(8^{\frac{2}{3}}-(-1)^{\frac{2}{3}}\right) \;=\; 3(4-1) \;=\; 9$$

$1\frac{1}{2}$ $2x^{-\frac{1}{3}}$ IS NOT CONTINUOUS @ $x=0 \in [-1,8]$ $\frac{1}{2}$

SO FTC PART 2 CAN'T BE USED

Find the following indefinite integrals. **SCORE: ______ / 8 PTS**

[a] $\int \frac{6x^2-5}{\sqrt[3]{1+10x-4x^3}}\,dx$.

$u = 1+10x-4x^3$ 1

$\frac{du}{dx} = 10-12x^2$

$-\frac{1}{2}du = (6x^2-5)dx$

$-\frac{1}{2}\int \frac{1}{\sqrt[3]{u}}du$ 1

$= -\frac{1}{2}\cdot\frac{3}{2}u^{\frac{2}{3}} + C$ 1

CAN BE SIMPLIFIED

$= -\frac{3}{4}(1+10x-4x^3)^{\frac{2}{3}} + C$

1 $\frac{1}{4}$

[b] Find $\int \operatorname{csch}^2 x \coth^4 x\,dx$.

$u = \coth x$ 1

$\frac{du}{dx} = -\operatorname{csch}^2 x$

$-du = \operatorname{csch}^2 x\,dx$

$-\int u^4 du$ 1

$= -\frac{1}{5}u^5 + C$ $\frac{1}{2}$

$= -\frac{1}{5}\coth^5 x + C$

1 $\frac{1}{4}$

Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x}\, dx$ by first multiplying the numerator and denominator of the integrand by $1+\sin x$. **SCORE: ______ / 5 PTS**

$$\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{1+\sin x}{1-\sin^2 x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{1+\sin x}{\cos^2 x}\, dx \quad 1$$

$$= \int_0^{\frac{\pi}{4}} (\sec^2 x + \sec x \tan x)\, dx \quad 1$$

$$= (\tan x + \sec x)\Big|_0^{\frac{\pi}{4}} \quad 2$$

$$= (1+\sqrt{2}) - (0+1) \quad \tfrac{1}{2}$$

$$= \sqrt{2} \quad \tfrac{1}{2}$$

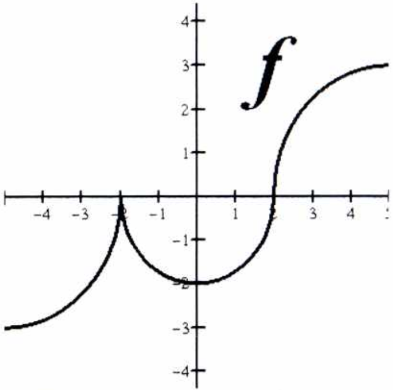
GIVE YOURSELF THIS $\frac{1}{2}$ POINT IF YOU SKIPPED THIS, BUT GOT RIGHT FINAL ANSWER

Let $g(x) = \int_5^x f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: ______ / 6 PTS**

[a] Find $g'(0)$. **Explain your answer very briefly.**

$g'(0) = f(0) = -2$

1 POINT EACH



[b] Find all intervals over which g is concave down. **Explain your answer very briefly.**

$g' = f$ IS DECREASING ON $[-2, 0]$

[c] Find the x – coordinates of all local minima of g. **Explain your answer very briefly.**

$g' = f$ CHANGES FROM NEGATIVE TO POSITIVE AT $x = 2$

If $f(x) = \int_4^{\sinh x} \sqrt{1+t^2}\, dt$ and $g(y) = \int_{-3}^{y} f(x)\, dx$, find $g''(x)$. SCORE: ______ / 4 PTS

For full credit, you must clearly show the use of all necessary properties of the definite integral.

$g'(x) = f(x)$

$g''(x) = f'(x) = \frac{d}{dx} \int_4^{\sinh x} \sqrt{1+t^2}\, dt$

1

$= \frac{d}{d \sinh x} \int_4^{\sinh x} \sqrt{1+t^2}\, dt \cdot \frac{d \sinh x}{dx}$ 1

$= $ 1 $\sqrt{1+\sinh^2 x} \cdot \cosh x$ $\frac{1}{2}$ $= \cosh^2 x$ $\frac{1}{2}$

A town decided to build a scenic path from its tourist center to its rose garden. If $C(l)$ = linear cost (in thousands of dollars per meter) of building the part of the path l meters from the tourist center, explain the meaning of the equation $\int_{200}^{600} G(h)\,dh = 100$. **SCORE: ______ / 2 PTS**

<u>In your explanation, give the meaning and units of all numbers in the equation.</u>

IT COST \$100,000 TO BUILD THE PART OF THE PATH BETWEEN 200m AND 600m FROM THE TOURIST CENTER

Answer the following questions about the definition of the definite integral as presented in lecture. **SCORE: ______ / 3 PTS**
(Your answers may refer to the fact that the definite integral equals the area under a curve which is above the $x-$axis.)

[a] Why is there a limit ($\lim$) in the definition and why does the index of the limit approach the value that it does ?

THE SUM OF THE AREAS OF THE RECTANGLES APPROACHES THE AREA UNDER THE CURVE AS EACH RECTANGLE GETS NARROWER AND THE NUMBER OF RECTANGLES APPROACHES ∞ (IE. $\lim_{n\to\infty}$)

[b] What is the difference between using $f(x_i^*)$ and $f(a+i\Delta x)$ in the definition ?

$f(a+i\Delta x)$ USES ENDPOINTS OF EACH SUBINTERVAL TO DETERMINE HEIGHTS OF RECTANGLES;
$f(x_i^*)$ USES ANY POINT IN EACH SUBINTERVAL