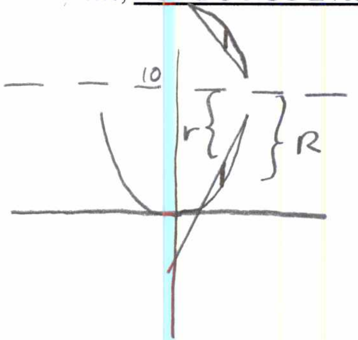
The region bounded by $y = x^2$ and $y = 4x - 3$ is revolved around the line $y = 10$. **SCORE: ______ / 9 PTS**

[a] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the disk or washer method.



$x^2 = 4x - 3 \longrightarrow x^2 - 4x + 3 = 0$

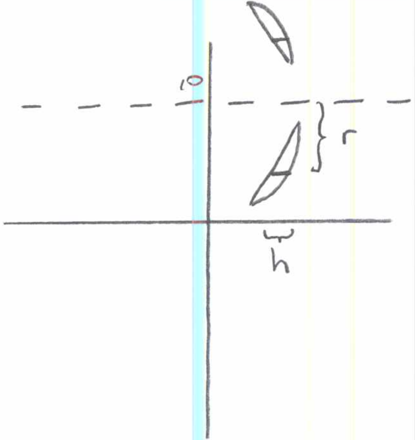
$x = 1, 3$

$\pi \int_1^3 ((10 - x^2)^2 - (10 - (4x-3))^2 \, dx$

$= \underbrace{\pi}_{\frac{1}{2}} \overbrace{\int_1^3}^{\frac{1}{2}} \underbrace{((10 - x^2)^2}_{\frac{1}{2}} \underbrace{-}_{\frac{1}{2}} \underbrace{(13 - 4x)^2}_{1}) \underbrace{dx}_{\frac{1}{2}}$

1

[b] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the shell method.



$x = 1, 3 \rightarrow y = 1, 9$

$y = x^2 \longrightarrow x = \sqrt{y}$ SINCE $x \geq 0$

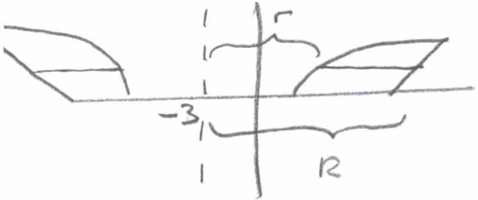
$y = 4x - 3 \longrightarrow x = \frac{1}{4}(y+3)$

$$2\pi \int_{\underbrace{1}_{\frac{1}{2}}}^{9} (10-y)\left(\sqrt{y} - \frac{1}{4}(y+3)\right) dy$$

$$= \underbrace{2\pi}_{\frac{1}{2}} \int_{\underbrace{1}_{\frac{1}{2}}}^{9} \underbrace{(10-y)}_{1}\underbrace{\left(-\frac{1}{4}y + \sqrt{y} - \frac{3}{4}\right)}_{1} \underbrace{dy}_{\frac{1}{2}}$$

The region bounded by $y=\sqrt{x-1}$, $y=x-3$ and $y=0$ is revolved around the line $x=-3$. **SCORE: ______ / 6 PTS**

Write, **BUT DO NOT EVALUATE**, a **SINGLE** integral for the volume of the solid.



$y=\sqrt{x-1} \longrightarrow x=y^2+1$

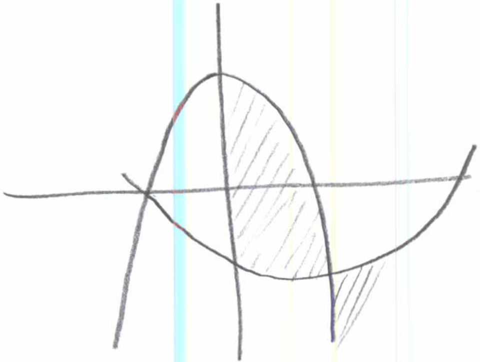
$y=x-3 \longrightarrow x=y+3$

$y^2+1=y+3 \longrightarrow y^2-y-2=0$

$y=2, \cancel{-1}$

$$\pi\int_0^2 \left((y+3-{}^-3)^2-(y^2+1-{}^-3)^2\right)dy = \pi\int_0^2\left((y+6)^2-(y^2+4)^2\right)dy$$

Find the area between the curves $y = 3 - 3x^2$ and $y = x^2 - 4x - 5$ over the interval $0 \le x \le 3$. **SCORE: ______ / 6 PTS**



$3 - 3x^2 = x^2 - 4x - 5 \rightarrow 4x^2 - 4x - 8 = 0$

$\rightarrow x^2 - x - 2 = 0$

$x = \cancel{-1}, 2$

$\int_0^2 (3 - 3x^2 - (x^2 - 4x - 5))\,dx$

$+ \int_2^3 (x^2 - 4x - 5 - (3 - 3x^2))\,dx$

$= \int_0^2 (8 + 4x - 4x^2)\,dx + \int_2^3 (4x^2 - 4x - 8)\,dx$ 3

$= (8x + 2x^2 - \frac{4}{3}x^3)\Big|_0^2 + (\frac{4}{3}x^3 - 2x^2 - 8x)\Big|_2^3 = \frac{40}{3} + \frac{22}{3} = \frac{62}{3}$ 1

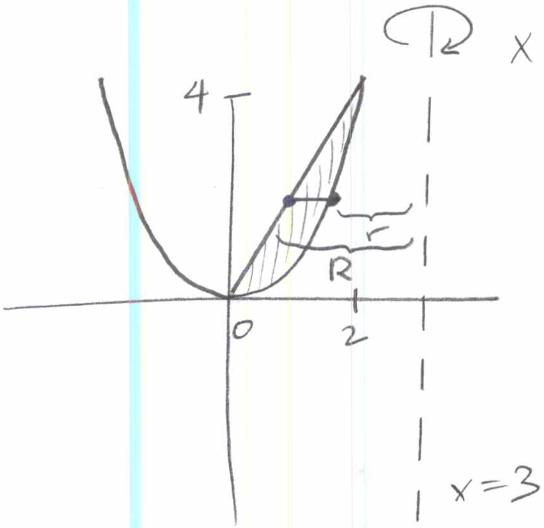
2

A solid is created by revolving a region around an axis of revolution. Sketch the region and find the equation of the axis of revolution if the volume of the solid is $\pi\int_0^4 ((3-\sqrt{y})^2-(3-\frac{1}{2}y)^2)\,dy$. ← ① BONUS POINT IF YOU REALIZED ORDER OF SUBTRACTION WAS REVERSED

SCORE: ______ / 4 PTS

$x=\sqrt{y} \to y=x^2$

$x=\frac{1}{2}y \to y=2x$



① FOR SKETCHING AXIS @ $x=3$

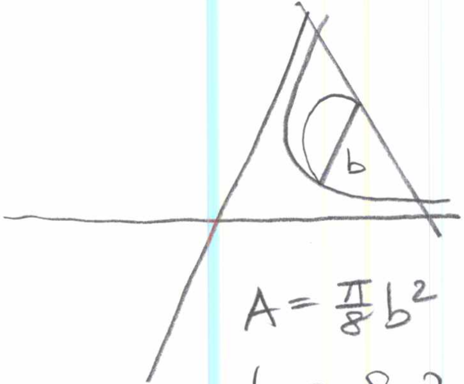
① " " $y=2x$

① " " $y=x^2$

① FOR SHADING CORRECT REGION

↑ OR SOMEHOW INDICATING

The base of a solid is the region bounded by $y = \frac{6}{x}$ and $y = 8 - 2x$. Cross sections perpendicular to the $x -$ axis **SCORE: ______ / 5 PTS** are semicircles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.



$\frac{6}{x} = 8 - 2x \rightarrow 6 = 8x - 2x^2$

$\rightarrow 2x^2 - 8x + 6 = 0$

$\rightarrow x^2 - 4x + 3 = 0$

$x = 1, 3$

$A = \frac{\pi}{8} b^2$

$b = 8 - 2x - \frac{6}{x}$

$$\underbrace{\frac{\pi}{8}}_{1} \int_{\underbrace{1}_{1}}^{\overbrace{3}^{1}} \underbrace{\left(8 - 2x - \frac{6}{x}\right)}_{1}{}^{\overbrace{2}^{\frac{1}{2}}} \underbrace{dx}_{\frac{1}{2}}$$