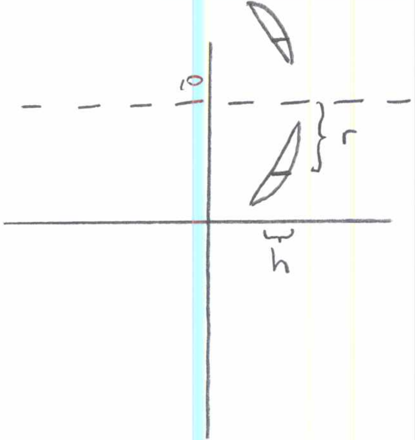
[b] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the shell method.



$x = 1, 3 \rightarrow y = 1, 9$

$y = x^2 \longrightarrow x = \sqrt{y}$ SINCE $x \geq 0$

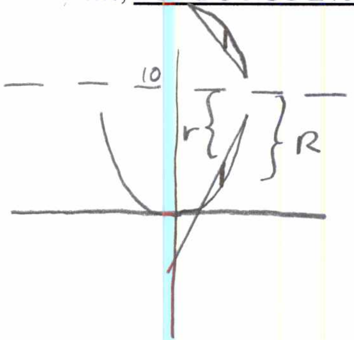
$y = 4x - 3 \longrightarrow x = \frac{1}{4}(y+3)$

$$2\pi \underbrace{\int_1^9}_{\frac{1}{2}} (10-y)\left(\sqrt{y} - \tfrac{1}{4}(y+3)\right) dy$$

$$= \underbrace{2\pi}_{\frac{1}{2}} \underbrace{\int_1^9}_{\frac{1}{2}} \underbrace{(10-y)}_{1}\underbrace{\left(-\tfrac{1}{4}y + \sqrt{y} - \tfrac{3}{4}\right)}_{1} \underbrace{dy}_{\frac{1}{2}}$$

The region bounded by $y = x^2$ and $y = 4x - 3$ is revolved around the line $y = 10$. **SCORE: ______ / 9 PTS**

[a] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the disk or washer method.



$$x^2 = 4x - 3 \longrightarrow x^2 - 4x + 3 = 0$$

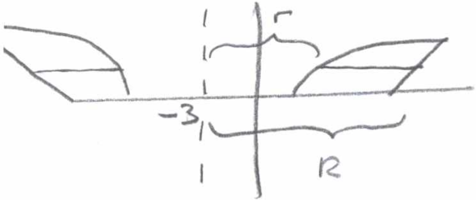
$$x = 1, 3$$

$$\pi \int_1^3 ((10 - x^2)^2 - (10 - (4x - 3))^2 \, dx$$

$$= \underbrace{\pi}_{\frac{1}{2}} \int_{\underbrace{1}_{\frac{1}{2}}}^{\overbrace{3}^{\frac{1}{2}}} (\underbrace{(10 - x^2)}_{\frac{1}{2}}\overbrace{{}^2 \underbrace{-}_{\frac{1}{2}} (\underbrace{13 - 4x}_{1})^2}^{1}) \underbrace{dx}_{\frac{1}{2}}$$

The region bounded by $y = \sqrt{x-1}$, $y = x-3$ and $y = 0$ is revolved around the line $x = -3$. **SCORE: ______ / 6 PTS**

Write, **<u>BUT DO NOT EVALUATE</u>**, a **<u>SINGLE</u>** integral for the volume of the solid.



$y = \sqrt{x-1} \rightarrow x = y^2 + 1$

$y = x - 3 \rightarrow x = y + 3$

$y^2 + 1 = y + 3 \rightarrow y^2 - y - 2 = 0$

$y = 2, \cancel{-1}$

$\pi \int_0^2 ((y+3 - {}^-3)^2 - (y^2 + 1 - {}^-3)^2)\, dy$ $= \pi \int_0^2 ((y+6)^2 - (y^2+4)^2)\, dy$

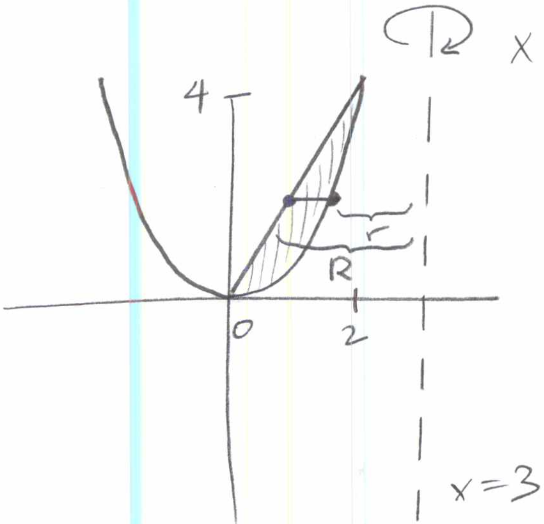
1 ½ ½ 1 1 ½ 1 ½ 1 ½

A solid is created by revolving a region around an axis of revolution. Sketch the region and find the equation of the axis of revolution if the volume of the solid is $\pi\int_0^4 ((3-\sqrt{y})^2 - (3-\frac{1}{2}y)^2)\,dy$. ← (1) BONUS POINT IF YOU REALIZED ORDER OF SUBTRACTION WAS REVERSED

SCORE: ______ / 4 PTS

$x=\sqrt{y} \rightarrow y=x^2$

$x=\frac{1}{2}y \rightarrow y=2x$



(1) FOR SKETCHING AXIS @ $x=3$

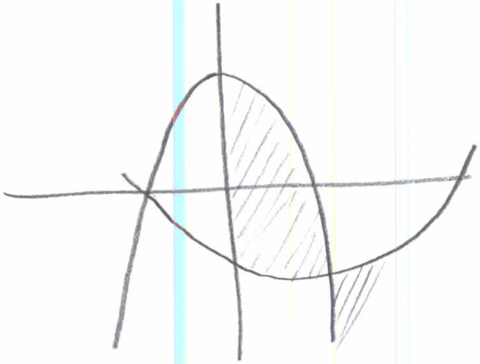
(1) " " $y=2x$

(1) " " $y=x^2$

(1) FOR SHADING CORRECT REGION

↑ OR SOMEHOW INDICATING

Find the area between the curves $y = 3 - 3x^2$ and $y = x^2 - 4x - 5$ over the interval $0 \le x \le 3$. **SCORE: ______ / 6 PTS**



$$3 - 3x^2 = x^2 - 4x - 5 \rightarrow 4x^2 - 4x - 8 = 0$$

$$\rightarrow x^2 - x - 2 = 0$$

$$x = \cancel{-1}, 2$$

$$\int_0^2 (3 - 3x^2 - (x^2 - 4x - 5))\,dx$$

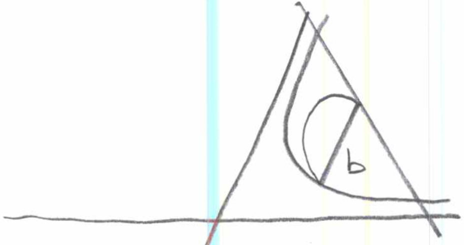
$$+ \int_2^3 (x^2 - 4x - 5 - (3 - 3x^2))\,dx$$

$$= \int_0^2 (8 + 4x - 4x^2)\,dx + \int_2^3 (4x^2 - 4x - 8)\,dx \quad 3$$

$$= 2 \; (8x + 2x^2 - \tfrac{4}{3}x^3)\Big|_0^2 + (\tfrac{4}{3}x^3 - 2x^2 - 8x)\Big|_2^3 = \frac{40}{3} + \frac{22}{3} = \frac{62}{3} \quad 1$$

The base of a solid is the region bounded by $y = \dfrac{6}{x}$ and $y = 8 - 2x$. Cross sections perpendicular to the $x-$axis **SCORE: ______ / 5 PTS**

are semicircles. Write, **<u>BUT DO NOT EVALUATE</u>**, an integral (or sum of integrals) for the volume of the solid.



$$\frac{6}{x} = 8 - 2x \rightarrow 6 = 8x - 2x^2$$

$$\rightarrow 2x^2 - 8x + 6 = 0$$

$$\rightarrow x^2 - 4x + 3 = 0$$

$$x = 1, 3$$

$$A = \frac{\pi}{8} b^2$$

$$b = 8 - 2x - \frac{6}{x}$$

$$\underbrace{\frac{\pi}{8}}_{1} \int_{\underbrace{1}_{1}}^{\overbrace{3}^{1}} \underbrace{\left(8 - 2x - \frac{6}{x}\right)}_{1}^{\overbrace{2}^{\frac{1}{2}}} \underbrace{dx}_{\frac{1}{2}}$$