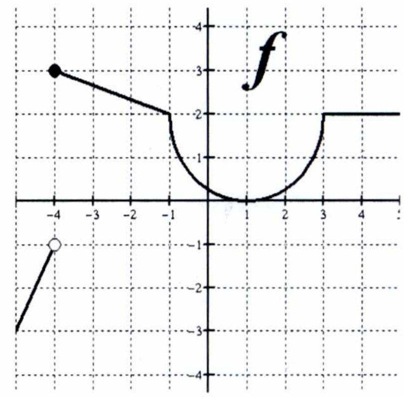
Math 1B Quiz 5
7:30am – 8:20am Version Z
Fri May 18, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ______________________

SCORE: _____ / 30 POINTS

NO CALCULATORS ALLOWED

The graph of f is shown on the right, and consists of 2 line segments, a semi-circle and another line segment. **SCORE: ______ / 3 PTS**
Find the average value of f on the interval $[-4, 3]$.
You must find the the exact value, not an approximation.



$$\frac{\int_{-4}^{3} f(x)\,dx}{3 - -4} = \frac{\frac{1}{2}\cdot 3\cdot(3+2) + 8 - \frac{1}{2}\cdot 4\pi}{7}$$

$$= \frac{\frac{15}{2} + 8 - 2\pi}{7} = \frac{\frac{31}{2} - 2\pi}{7} \; 1 \; 1 = \frac{31}{14} - \frac{2\pi}{7} \; 1$$

Find the average value of $f(x) = \dfrac{1+x^2}{x}$ on the interval $[1, 6]$. **SCORE: ______ / 4 PTS**

$$\frac{1}{5}\int_1^6 \frac{1+x^2}{x}\,dx = \frac{1}{5}\int_1^6 \left(\frac{1}{x} + x\right)dx = \frac{1}{5}\left(\ln|x| + \frac{1}{2}x^2\right)\Big|_1^6 \quad 1 \; 1 \; 1$$

$$= \frac{1}{5}\left(\ln 6 + 18 - \left(\ln 1 + \frac{1}{2}\right)\right)$$

$$= \frac{1}{5}\left(\ln 6 - \frac{35}{2}\right) = \frac{1}{5}\ln 6 - \frac{7}{2} \quad 1$$

Find the length of the curve $y = \cosh^{-1} x$ on the interval $2 \le x \le 3$. **SCORE: ______ / 7 PTS**
Your final answer must NOT involve hyperbolic NOR inverse hyperbolic functions. Simplify your final answers.

$$\int_2^3 \sqrt{1 + \left(\frac{1}{\sqrt{x^2-1}}\right)^2}\,dx$$

$$= \int_2^3 \sqrt{1 + \frac{1}{x^2-1}}\,dx \quad 3$$

$$= \int_2^3 \sqrt{\frac{x^2}{x^2-1}}\,dx$$

$$= \int_2^3 \frac{x}{\sqrt{x^2-1}}\,dx \quad 1$$

$$u = x^2 - 1$$

$$\frac{1}{2}du = x\,dx$$

$$= \frac{1}{2}\int_3^8 u^{-\frac{1}{2}}\,du \quad 1$$

$$= \frac{1}{2}\cdot 2u^{\frac{1}{2}}\Big|_3^8 \quad 1$$

OK IF SIMPLIFIED TO $u^{\frac{1}{2}}$

$$= \sqrt{8} - \sqrt{3}$$

$$= 2\sqrt{2} - \sqrt{3} \quad 1$$

OR **ALTERNATE SOLUTION**

$x = \cosh y \qquad \cosh^{-1} 2 \le y \le \cosh^{-1} 3$

$$\underbrace{\int_{\cosh^{-1} 2}^{\cosh^{-1} 3} \sqrt{1 + \sinh^2 y}\, dy}_{3}$$

$$= \int_{\cosh^{-1} 2}^{\cosh^{-1} 3} \sqrt{\cosh^2 y}\, dy$$

$$= \int_{\cosh^{-1} 2}^{\cosh^{-1} 3} \underbrace{\cosh y}_{1}\, dy$$

$$= \underbrace{\sinh y}_{1} \Big|_{\cosh^{-1} 2}^{\cosh^{-1} 3}$$

$$= \sinh \cosh^{-1} 3 - \sinh \cosh^{-1} 2$$

$$= \sqrt{8} - \sqrt{3}$$

$$= \underbrace{2\sqrt{2} - \sqrt{3}}_{2}$$

TO FIND $\sinh \cosh^{-1} c$:

$z = \cosh^{-1} c \Rightarrow z \geq 0$

$\cosh z = c$

$\cosh^2 z - 1 = c^2 - 1$

$\sinh^2 z = c^2 - 1$

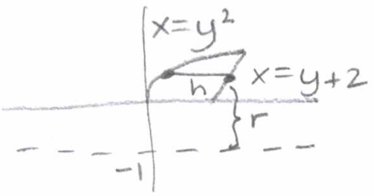
$\sinh z = \pm\sqrt{c^2 - 1}$

$\sinh z \geq 0$ SINCE $z \geq 0$

SO, $\sinh z = \sqrt{c^2 - 1}$

SO, $\sinh \cosh^{-1} c = \sqrt{c^2 - 1}$

The region bounded by $y=\sqrt{x}$, $y=x-2$ and $y=0$ is revolved around the line $y=-1$. **SCORE: ______ / 8 PTS**
Find the volume of the solid. **Your final answer must be a number, not an integral.**



$y^2=y+2$

$y^2-y-2=0$

$y=-1,2$

$$2\pi\int_0^2 (y+1)(y+2-y^2)\,dy$$

$$=2\pi\int_0^2 (y^2+2y-y^3+y+2-y^2)\,dy$$

$$=2\pi\int_0^2 (-y^3+3y+2)\,dy$$

$$=2\pi\left(-\tfrac{1}{4}y^4+\tfrac{3}{2}y^2+2y\right)\Big|_0^2$$

$$=2\pi\left(-\tfrac{1}{4}\cdot 16+\tfrac{3}{2}\cdot 4+2\cdot 2\right)$$

$$=2\pi(-4+6+4)$$

$$=12\pi$$

A trendy restaurant is trying a new reservation model. On June 1, they will open the automated online reservation system from midnight until 6 am, for customers to book and prepay seats for July. Let X be the fraction of all available July seats which are booked using this model. Based on prior experience at similar restaurants, the probability density function has the form $f(x)=kx^3(1-x)$ for some constant k, for $x\in[0,1]$. **SCORE: ______ / 8 PTS**

[a] Find the value of k.

$$\int_0^1 kx^3(1-x)\,dx=1 \Rightarrow k\int_0^1 (x^3-x^4)\,dx=1$$

$$k\left(\tfrac{1}{4}x^4-\tfrac{1}{5}x^5\right)\Big|_0^1=1$$

$$k\left(\tfrac{1}{4}-\tfrac{1}{5}\right)=1$$

$$\frac{k}{20}=1 \Rightarrow k=20$$

[b] What is the probability that at least half of all July seats will be booked using this model ?

$$\int_{\frac{1}{2}}^1 20x^3(1-x)\,dx=\int_{\frac{1}{2}}^1 (20x^3-20x^4)\,dx=(5x^4-4x^5)\Big|_{\frac{1}{2}}^1$$

$$=5\left(1-\tfrac{1}{16}\right)-4\left(1-\tfrac{1}{32}\right)=\frac{75}{16}-\frac{31}{8}=\frac{13}{16}$$

OR

OR $$1-\int_0^{\frac{1}{2}} 20x^3(1-x)\,dx=1-(5x^4-4x^5)\Big|_0^{\frac{1}{2}}=1-\left(\tfrac{5}{16}-\tfrac{1}{8}\right)=1-\frac{3}{16}=\frac{13}{16}$$

[c] What is the expected fraction of July seats which will be booked using this model ?

$$\int_0^1 x\cdot 20x^3(1-x)\,dx=\int_0^1 20x^4(1-x)\,dx$$

$$=\int_0^1 (20x^4-20x^5)\,dx$$

$$=\left(4x^5-\tfrac{10}{3}x^6\right)\Big|_0^1=4-\frac{10}{3}=\frac{2}{3}$$