Math 1B Quiz 6
7:30am – 8:20am Version Q
Fri Jun 1, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ______________________

SCORE: _____ / 30 POINTS

NO CALCULATORS ALLOWED

Using the integral based definition of $\ln(x)$, prove that $\ln(x^r) = r\ln(x)$ using the substitution method shown in lecture. **SCORE: _____ / 7 PTS**

$r\ln(x)$

$= \underbrace{r\int_1^x \frac{1}{t}\,dt}_{1}$ Let $\underbrace{u = t^r}_{2}$ so $\underbrace{du = rt^{r-1}\,dt}_{1}$ or $dt = \frac{du}{rt^{r-1}}$ and $\frac{1}{t}\,dt = \frac{1}{t}\frac{du}{rt^{r-1}} = \frac{du}{rt^r} = \frac{du}{ru}$

and $t=1 \Rightarrow u = 1^r = 1$ and $t = x \Rightarrow u = x^r$

1

$= r\int_1^{x^r} \underbrace{\frac{du}{ru}}_{1} = \underbrace{\int_1^{x^r} \frac{du}{u}}_{1} = \ln(x^r)$

GRADE AGAINST ONLY 1 VERSION

ALTERNATE SOLUTION

$\ln(x^r)$

$= \underbrace{\int_1^{x^r} \frac{1}{t}\,dt}_{1}$ Let $\underbrace{u = t^{\frac{1}{r}}}_{2}$ so $\underbrace{du = \frac{1}{r}t^{\frac{1}{r}-1}\,dt}_{1}$ or $dt = \frac{r\,du}{t^{\frac{1}{r}-1}}$ and $\frac{1}{t}\,dt = \frac{1}{t}\frac{r\,du}{t^{\frac{1}{r}-1}} = \frac{r\,du}{t^{\frac{1}{r}}} = \frac{r\,du}{u}$

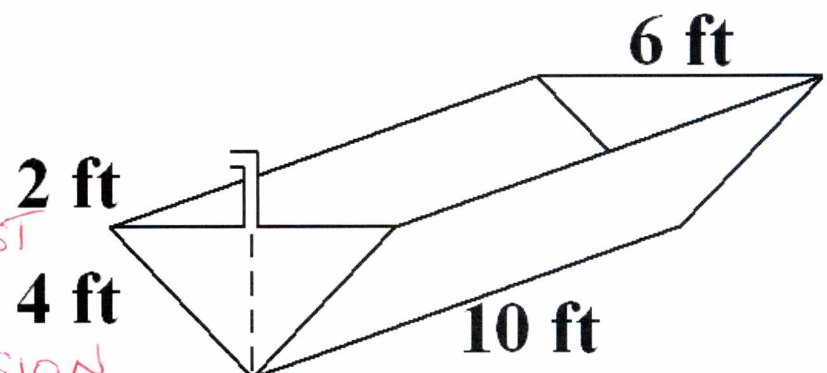
1

and $t=1 \Rightarrow u = 1^{\frac{1}{r}} = 1$ and $t = (x^r)^{\frac{1}{r}} \Rightarrow u = x$

$= \int_1^x \underbrace{\frac{r\,du}{u}}_{1} = \underbrace{r\int_1^x \frac{1}{u}\,du}_{1} = r\ln(x)$

A tank in the shape of the triangular prism shown on the right is filled with water. **SCORE: _____ / 7 PTS**
Write, **BUT DO NOT EVALUATE**, an integral for the work required to pump the water out of the spout.
NOTE: The spout is 2 feet tall. The tank is 4 feet tall, 6 feet wide across the top, and 10 feet long.

Your solution will depend on the scale you used
Pick the solution that matches the values of x you used
For the bottom & top of the triangle, and the top of the spout



GRADE AGAINST ONLY 1 VERSION

If your scale was
$x = 0$ at bottom of triangle
$x = 4$ at top of triangle / bottom of spout
$x = 6$ at top of spout

$1 \quad \int_0^4 \underbrace{62.5}_{\frac{1}{2}}\left(\underbrace{10}_{1}\times\underbrace{\frac{3}{2}x}_{2}\right)\underbrace{(6-x)}_{2}\,\underbrace{dx}_{\frac{1}{2}}$

If your scale was
$x = 4$ at bottom of triangle
$x = 0$ at top of triangle / bottom of spout
$x = -2$ at top of spout

$1 \quad \int_0^4 \underbrace{62.5}_{\frac{1}{2}}\left(\underbrace{10}_{1}\times\underbrace{\frac{3}{2}(4-x)}_{2}\right)\underbrace{(x+2)}_{2}\,\underbrace{dx}_{\frac{1}{2}}$

If your scale was
$x = 6$ at bottom of triangle
$x = 2$ at top of triangle / bottom of spout
$x = 0$ at top of spout

$1 \quad \int_2^6 \underbrace{62.5}_{\frac{1}{2}}\left(\underbrace{10}_{1}\times\underbrace{\frac{3}{2}(6-x)}_{2}\right)\underbrace{(x)}_{2}\,\underbrace{dx}_{\frac{1}{2}}$

If you used any other scale come talk to me

OK IF YOU SAID $10\times\frac{3}{2} = 15$

Evaluate the following indefinite integrals. **SCORE: _____ / 16 PTS**

[a] $\int \frac{\ln x}{x^3}\,dx = \int x^{-3}\ln x\,dx$

$u = \ln x \qquad dv = x^{-3}\,dx$

$du = \frac{1}{x}\,dx \qquad v = -\frac{1}{2}x^{-2}$

$\int x^{-3}\ln x\,dx$

$= -\frac{1}{2}x^{-2}\ln x + \frac{1}{2}\int x^{-3}\,dx$

$= \underbrace{-\frac{1}{2}x^{-2}\ln x}_{1\frac{1}{2}} \underbrace{-\frac{1}{4}x^{-2}}_{2} \underbrace{+C}_{\frac{1}{2}}$

$= -\frac{1}{4}x^{-2}(1+2\ln x)+C$

[b] $\int \frac{x}{3e^{3x}}\,dx = \frac{1}{3}\int xe^{-3x}\,dx$

$u = x \qquad dv = e^{-3x}\,dx$

$du = dx \qquad v = -\frac{1}{3}e^{-3x}$

$\frac{1}{3}\int xe^{-3x}\,dx$

$= \frac{1}{3}\left(-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx\right)$

$= \frac{1}{3}\left(-\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}\right)+C$

$= \underbrace{-\frac{1}{9}xe^{-3x}}_{1\frac{1}{2}} \underbrace{-\frac{1}{27}e^{-3x}}_{2} \underbrace{+C}_{\frac{1}{2}}$

$= -\frac{1}{27}e^{-3x}(3x+1)+C$

[c] $\int x\sin 3x\,dx$

$u = x \qquad dv = \sin 3x\,dx$

$du = dx \qquad v = -\frac{1}{3}\cos 3x$

$\int x\sin 3x\,dx$

$= -\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x\,dx$

$= \underbrace{-\frac{1}{3}x\cos 3x}_{1\frac{1}{2}} \underbrace{+\frac{1}{9}\sin 3x}_{2} \underbrace{+C}_{\frac{1}{2}}$

[d] $\int 3x\sec^2 3x\,dx$

$u = x \qquad dv = 3\sec^2 3x\,dx$

$du = dx \qquad v = \tan 3x$

$\int 3x\sec^2 3x\,dx$

$= x\tan 3x + \int \tan 3x\,dx$

$= x\tan 3x + \frac{1}{3}\ln|\cos 3x| + C$

$= \underbrace{x\tan 3x}_{1\frac{1}{2}} \underbrace{-\frac{1}{3}\ln|\sec 3x|}_{2} \underbrace{+C}_{\frac{1}{2}}$