Math 1B Quiz 7
7:30am – 8:20am Version B
Fri Jun 8, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ______________________

SCORE: _____ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du \quad \text{and} \quad \int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du \quad (n \neq 0)$$

$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2}u\tan u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du \quad (n \neq 1)$$

NO CALCULATORS ALLOWED

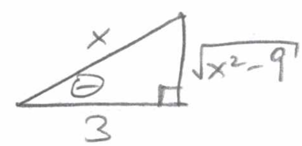
Evaluate $\int \frac{x^2}{\sqrt{x^2-9}}\,dx$. Simplify your final answer. **SCORE: _____ / ~~8~~ 9 PTS**

$$x^2 - 9 = 9\left(\frac{x^2}{9} - 1\right) = 9(\sec^2\theta - 1) = 9\tan^2\theta$$

$$\frac{x^2}{9} = \sec^2\theta \rightarrow \frac{x}{3} = \sec\theta \rightarrow x = 3\sec\theta \quad (1)$$

$$dx = 3\sec\theta\tan\theta\,d\theta$$

$$\int \frac{9\sec^2\theta}{3\tan\theta} \cdot 3\sec\theta\tan\theta\,d\theta \quad (1)\ (1)\ (1)$$



$$= 9\int \sec^3\theta\,d\theta \quad (\tfrac{1}{2})$$

$$= \frac{9}{2}\sec\theta\tan\theta + \frac{9}{2}\ln|\sec\theta + \tan\theta| + C \quad (1)\ (1)$$

$$= \frac{9}{2}\,\frac{x}{3}\,\frac{\sqrt{x^2-9}}{3} + \frac{9}{2}\ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C = \frac{1}{2}x\sqrt{x^2-9} + \frac{9}{2}\ln\left|x + \sqrt{x^2-9}\right| + C \quad (\tfrac{1}{2})\ (1)\ (\tfrac{1}{2})\ (\tfrac{1}{2})$$

Use integration by parts to prove the reduction formula for $\int \cos^n x\,dx$ at the top of this quiz. **SCORE: _____ / 6 PTS**

$u = \cos^{n-1}x \qquad dv = \cos x\,dx$

$du = -(n-1)\cos^{n-2}x\sin x \qquad v = \sin x\,dx$

$$\int \cos^n x\,dx = \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x\sin^2 x\,dx \quad (1)\ (1)$$

$$= \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x(1-\cos^2 x)\,dx \quad (1)$$

$$= \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\int \cos^n x\,dx$$

$$n\int \cos^n x\,dx = \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x\,dx \quad (2)$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx \quad (1)$$

Evaluate $\int x^3 (\ln x)^2 \, dx$. **Simplify your final answer.** SCORE: ____ / 5 PTS

$u = (\ln x)^2 \quad dv = x^3 dx$

$du = \frac{2\ln x}{x} dx \quad v = \frac{1}{4}x^4$

☆ SEE ALTERNATE SOLUTION IF YOU USED TABLE METHOD

(1) $\frac{1}{4}x^4(\ln x)^2 - \int \frac{1}{2}x^3 \ln x \, dx$ (1)

$u = \ln x \quad dv = \frac{1}{2}x^3$

$du = \frac{1}{x}dx \quad v = \frac{1}{8}x^4$

$= \frac{1}{4}x^4(\ln x)^2 - \left[\frac{1}{8}x^4 \ln x - \int \frac{1}{8}x^3 dx\right]$ (1) (1)

$= \frac{1}{4}x^4(\ln x)^2 - \frac{1}{8}x^4 \ln x + \frac{1}{32}x^4 + C$ $(\frac{1}{2})$ $(\frac{1}{2})$

Evaluate $\int e^{-2x} \sin 4x \, dx$. **Simplify your final answer.** SCORE: ____ / 5 PTS

u	dv
$\sin 4x$	$+\ e^{-2x}$
$4\cos 4x$	$-\ -\frac{1}{2}e^{-2x}$
$-16\sin 4x$	$+\ \frac{1}{4}e^{-2x}$

← SEE ALTERNATE SOLUTION IF YOU SWITCHED u, dv ☆

$\int e^{-2x}\sin 4x \, dx = -\frac{1}{2}e^{-2x}\sin 4x - e^{-2x}\cos 4x - 4\int e^{-2x}\sin 4x \, dx$ (1) (1) (1)

$5\int e^{-2x}\sin 4x \, dx = -\frac{1}{2}e^{-2x}\sin 4x - e^{-2x}\cos 4x$ $(\frac{1}{2})$

$\int e^{-2x}\sin 4x \, dx = -\frac{1}{10}e^{-2x}\sin 4x - \frac{1}{5}e^{-2x}\cos 4x + C$ $(\frac{1}{2})$

(1)

Evaluate $\int \tan^4 x \sec^6 x \, dx$. **Simplify your final answer.** SCORE: ____ / 5 PTS

$u = \tan x$ (1)

$du = \sec^2 x \, dx$

TALK TO ME IF YOU USED REDUCTION FORMULAE (LONG) ☆

$\tan^4 x \sec^6 x \, dx = \tan^4 x \sec^4 x \cdot \sec^2 x \, dx$

$= u^4(u^2+1)^2 du$

$\int u^4(u^2+1)^2 du = \int u^4(u^4+2u^2+1) du$

(1)

$= \int (u^8 + 2u^6 + u^4) du$ $(\frac{1}{2})$

$=$ (1) $\frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5 + C$

$=$ (1) $\frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x + C$ $(\frac{1}{2})$