

SCORE: ____ / 30 POINTS

You may or may not need the following reduction formulae.

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du \quad \text{and} \quad \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du \quad (n \neq 0)$$

$$\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du \quad (n \neq 1)$$

NO CALCULATORS ALLOWED

Evaluate $\int \frac{26}{x^3 + 6x^2 + 13x} dx$.

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$$x^3 + 6x^2 + 13x = x(x^2 + 6x + 13)$$

$$= x((x+3)^2 + 4)$$

$$= \int \left(\frac{26}{x} - \frac{2x+6}{x^2+6x+13} - \frac{6}{(x+3)^2+4} \right) dx$$

$$= 2 \ln|x| - \ln(x^2+6x+13) - 3 \tan^{-1} \frac{x+3}{2} + C$$

$$\frac{A}{x} + \frac{B(2x+6)+C}{x^2+6x+13} = \frac{26}{x^3+6x^2+13x}$$

$$A(x^2+6x+13) + Bx(2x+6) + Cx = 26$$

$$x=0: 13A = 26 \rightarrow A=2$$

$$x=-3: 4A-3C=26 \rightarrow 8-3C=26 \rightarrow C=-6$$

$$\text{COEF OF } x^2: A+2B=0 \rightarrow 2+2B=0 \rightarrow B=-1$$

SANITY CHECK:

$$x=2 \quad \frac{26}{8+24+26} = \frac{26}{58} = \frac{13}{29}$$

$$\frac{2}{2} + \frac{-1(10)+-6}{4+12+13} = 1 - \frac{16}{29} = \frac{13}{29} \quad \checkmark$$

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Evaluate $\int \frac{1-3x^3}{x^2+4x+4} dx$.

$$x^2+4x+4 \overline{) -3x^3+0x^2+0x+1}$$

$$\underline{-3x^3-12x^2-12x}$$

$$12x^2+12x+1$$

$$\underline{12x^2+48x+48}$$

$$\underline{-36x-47}$$

$$\frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{-36x-47}{(x+2)^2}$$

$$A(x+2) + B = -36x-47$$

$$x=-2: B = -25$$

$$\text{COEF OF } x: A = -36$$

$$\int \left(-36x+12 - \frac{36}{x+2} + \frac{25}{(x+2)^2} \right) dx$$

$$= -\frac{36}{2}x^2 + 12x - 36 \ln|x+2| - \frac{25}{x+2} + C$$

SANITY CHECK:

$$x=2 \quad \frac{1-3(8)}{4+8+4} = \frac{-23}{16}$$

$$-3(2)+12 - \frac{36}{4} + \frac{25}{16}$$

$$= -3 + \frac{25}{16} = \frac{-23}{16} \quad \checkmark$$

Evaluate $\int (1 + \sqrt[3]{x})^{12} dx$.

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① $u = 1 + \sqrt[3]{x}$
 $x = (u-1)^3$
 $dx = 3(u-1)^2 du$

① $\int u^{12} \cdot 3(u-1)^2 du = 3 \int u^{12} (u^2 - 2u + 1) du$

② $= 3 \int (u^{14} - 2u^{13} + u^{12}) du$

① $= 3 \left(\frac{1}{15} u^{15} - \frac{2}{14} u^{14} + \frac{1}{13} u^{13} \right) + C$

① $= \frac{1}{5} (1 + \sqrt[3]{x})^{15} - \frac{3}{7} (1 + \sqrt[3]{x})^{14} + \frac{3}{13} (1 + \sqrt[3]{x})^{13} + C$ ②

Evaluate $\int \frac{dx}{e^{2x} - 1}$.

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① $u = e^{2x} - 1$
 $du = 2e^{2x} dx = 2(u+1) dx$
 $dx = \frac{du}{2(u+1)}$

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$\int \left(\frac{1}{e^x - 1} - \frac{1}{e^x + 1} \right) dx$

$\frac{1}{2} \int \frac{1}{u+1} du = \frac{1}{2} \int \left(\frac{1}{u} - \frac{1}{u+1} \right) du$ ①

② $= \frac{1}{2} (\ln |u| - \ln |u+1|) + C$

② $= \frac{1}{2} (\ln |e^{2x} - 1| - \ln |e^{2x}|) + C$

$= \frac{1}{2} \ln |e^{2x} - 1| - x + C$ ②

Evaluate $\int_1^3 \frac{1}{(2x-3)^3} dx$. Show your work properly, using the correct notation shown in lecture.

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$= \int_1^{\frac{3}{2}} \frac{1}{(2x-3)^3} dx + \int_{\frac{3}{2}}^3 \frac{1}{(2x-3)^3} dx$ ①

★ $\rightarrow = \lim_{N \rightarrow \frac{3}{2}^-} \int_1^N \frac{1}{(2x-3)^3} dx$ ②

YOU MUST USE LIMIT NOTATION PROPERLY

$= \lim_{N \rightarrow \frac{3}{2}^-} \left[-\frac{1}{4} (2x-3)^{-2} \right]_1^N$ ①

$= \lim_{N \rightarrow \frac{3}{2}^-} \left[-\frac{1}{4} [(2N-3)^{-2} - 1] \right]$ ②

$= -\infty$ ② DIVERGES ②

① OR ②
 $\lim_{N \rightarrow \frac{3}{2}^+} \int_1^N \frac{1}{(2x-3)^3} dx$
 $= \lim_{N \rightarrow \frac{3}{2}^+} \left[-\frac{1}{4} (2x-3)^{-2} \right]_1^N$ ①
 $= \lim_{N \rightarrow \frac{3}{2}^+} \left[-\frac{1}{4} \left[\frac{1}{9} - (2x-3)^{-2} \right] \right]$ ②
 $= \infty$ ② DIVERGES ②