WHEN DOING VOLUME OF REVOLUTION PROBLEMS,
HOW DO YOU DECIDE WHICH WAY TO CUT THE SHAPE, AND
WHICH METHOD (DISK / WASHER / SHELL) TO USE TO SET UP THE INTEGRAL ?

[1] **How do you decide which direction to cut the region into rectangles ?**

NOTE: a boundary is an edge of the base region defined by a single equation

Overriding criteria (these criteria require you to cut in a specific direction)

    If there is a boundary which you cannot solve as $x = g(y)$ or $y = k$, you MUST cut vertically (dx)

    If there is a boundary which you cannot solve as $y = f(x)$ or $x = c$, you MUST cut horizontally (dy)

Preferred criteria (if the overriding criteria do not require you to cut in a specific direction):

    If possible, cut so that you never cross the perimeter more than twice with any cut
        to avoid getting washers within washers, or shells within shells

    If possible, cut so that you never cross a single boundary more than once with any cut
        to avoid having to convert a function in one variable into 2 functions in the other variable

    If possible, cut so that you never cross different boundaries with different cuts
        to avoid having to write multiple integrals

    If possible, cut so that the integrand is simpler
        to avoid having to find the anti-derivatives of complex functions

[2] **How do you find the method of integration after you've decided which direction to cut ?**

If the cut is parallel to the axis of revolution

    use the shell method

If the cut is perpendicular to the axis of revolution

    if the axis of revolution is a boundary of the region

        use the disk method

    if the axis of revolution is not a boundary of the region

        use the washer method

Volume for revolved solids (using discs/washer)

Draw the region to be revolved, the axis of revolution, and the reflection of the region over the axis

Cut the original region perpendicular to the axis of revolution and connect the ends of the cut to their reflections using circles

Find the area of the cross section

πr^2 if the axis of revolution is a boundary of the original region
where r is the distance from the axis to the opposite boundary

$\pi(R^2 - r^2)$ if the axis of revolution is not a boundary of the original region
where R is the distance from the axis to the farther endpoint of the cut
and r is the distance from the axis to the closer endpoint of the cut

Find the least (a) and greatest (b) values of the variable (look at the original region)

Integrate the area formula from a to b

Volume for revolved solids (using shells)

Draw the region to be revolved, the axis of revolution, and the reflection of the region over the axis

Cut the original region parallel to the axis of revolution and connect the ends of the cut to their reflections using circles

Find the radius (r) (from the axis of revolution to the cut) and the height (h) (the length of the cut) of the shell

Find the least (a) and greatest (b) values of the variable (look at the original region)

Integrate $2\pi rh$ from a to b

Volume for non-revolved solids with fixed shape cross sections

Draw the base region

Cut the region in the direction specified

Draw the cross section

Find a formula for the area of the cross section in terms of the length of the cut (s)

Find a formula for the length of the cut in terms of x (if the cut was perpendicular to the x-axis) or y (if the cut was perpendicular to the y-axis)

Substitute for s in the area formula to get a new area formula in terms of x or y

Find the least (a) and greatest (b) values of the remaining variable (look at the base region)

Integrate the area formula from a to b